少数の離散的な時系列を使ってフーリエ級数について係数の計算例を示す。下表はある都市の月別平均気温である。

月	1	2	3	4	5	6	7	8	9	10	11	12
平均気温	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

この $N = 12$ 個の等間隔に与えられた離散時系列 $\{x(n), n = 1, 2, \cdots, 12\}$ に対して、正弦関数と余弦関数の振幅を適切に調整することによってそれらの和

$$x(n) = a_0 + \sum_{k-1}^{6} a_k \cos 2\pi f_k n + \sum_{k-1}^{5} b_k \sin 2\pi f_k n$$

が元の時系列の各点を通るようにすることを考える（$\Delta t = 1$）。ここで、$f_k = k/12$ は周波数である。上の時系列の場合、データ個数 N は 12 個であるから偶数の場合に当たり、式（1.16)、(1.17) 等を使って計算すると下表のようになる。

k	余弦係数 a_k	正弦係数 b_k
1	−5.30	−3.82
2	0.05	0.17
3	0.10	0.50
4	−0.52	−0.52
5	0.09	−0.58
6	−0.36	-

This is a sample of very small number of sampled time series. The following table shows the monthly average temperature of a city.

Month 1 2 …
Average Temperature (C) 3.4 4.5 …

For this equally discrete series x(n), n=1,2,…,12), give the following expression, where dt=1 and fk is frequency and equal to k/12.

X(n) = a0 + sigma(….) + sigma(…)

Obtain the coefficients so as to this function satisfies the given sampled data as close as possible.

The answer is given in the following table.

k	Cosine coefficients ak	Sine coefficients bk
1	0.05	0.17
…..		

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n=月	x(n)=平均気温	2π*(n-1)/12
1	3.4	0
2	4.5	0.523598776
3	4.3	1.047197551
4	8.7	1.570796327
5	13.3	2.094395102
6	13.8	2.617993878
7	16.1	3.141592654
8	15.5	3.665191429
9	14.1	4.188790205
10	8.9	4.71238898
11	7.4	5.235987756
12	3.6	5.759586532
	Given Date	
	a0	9.466666667

$$a(k) = -\frac{2}{n}\sum_{j=1}^{n} xj \cos(2\pi(j-1)k/n)$$

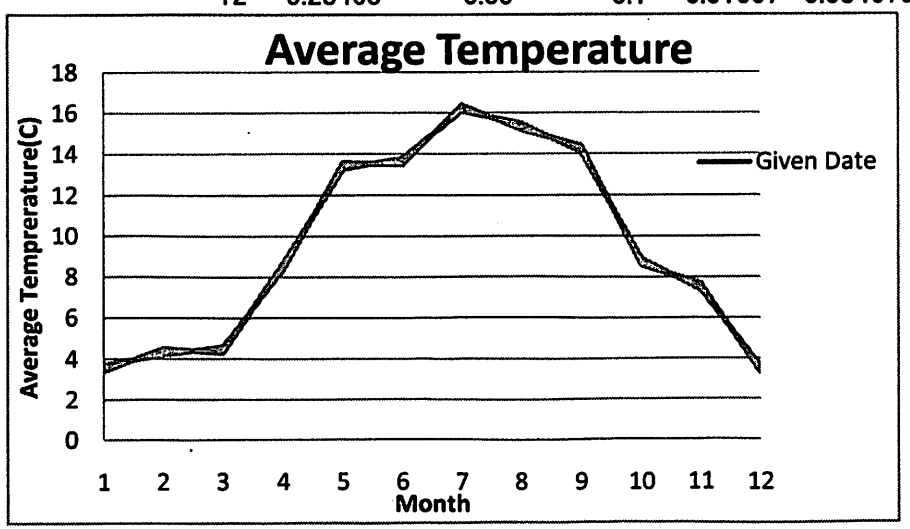
$$b(k) = -\frac{2}{n}\sum_{j=1}^{n} xj \sin(2\pi(j-1)k/n)$$

$$a(0) = -\frac{1}{n}\sum_{j=1}^{n} xj$$

$$xj = a(0) + \sum_{k=1}^{n/2} a(k) \cos(2\pi k(j-1)/n) + \sum_{k=1}^{n/2-1} b(k) \sin(2\pi k(j-1)/n)$$

k	ak	bk
1	-6.48496	-0.66292
2	0.175	0.043301
3	0.5	-0.1
4	-0.19167	0.707254
5	-0.36504	0.462917
6	0.6	1.78E-15

k	1	2	3	4	5	6	
余弦係数a	-6.48496	0.175	0.5	-0.19167	-0.36504	0.6	
正弦係数b	-0.66292	0.043301	-0.1	0.707254	0.462917	0	
n							Fourie series
1	-6.48496	0.175	0.5	-0.19167	-0.36504	0.6	3.7
2	-5.9476	0.125	-0.1	0.708333	0.547595	-0.6	4.2
3	-3.81658	-0.05	-0.5	-0.51667	-0.58342	0.6	4.6
4	-0.66292	-0.175	0.1	-0.19167	0.462917	-0.6	8.4
5	2.668376	-0.125	0.5	0.708333	-0.21838	0.6	13.6
6	5.284679	0.05	-0.1	-0.51667	-0.08468	-0.6	13.5
7	6.484956	0.175	-0.5	-0.19167	0.365044	0.6	16.4
8	5.947595	0.125	0.1	0.708333	-0.5476	-0.6	15.2
9	3.816581	-0.05	0.5	-0.51667	0.583419	0.6	14.4
10	0.662917	-0.175	-0.1	-0.19167	-0.46292	-0.6	8.6
11	-2.66838	-0.125	-0.5	0.708333	0.218376	0.6	7.7
12	-5.28468	0.05	0.1	-0.51667	0.084679	-0.6	3.3



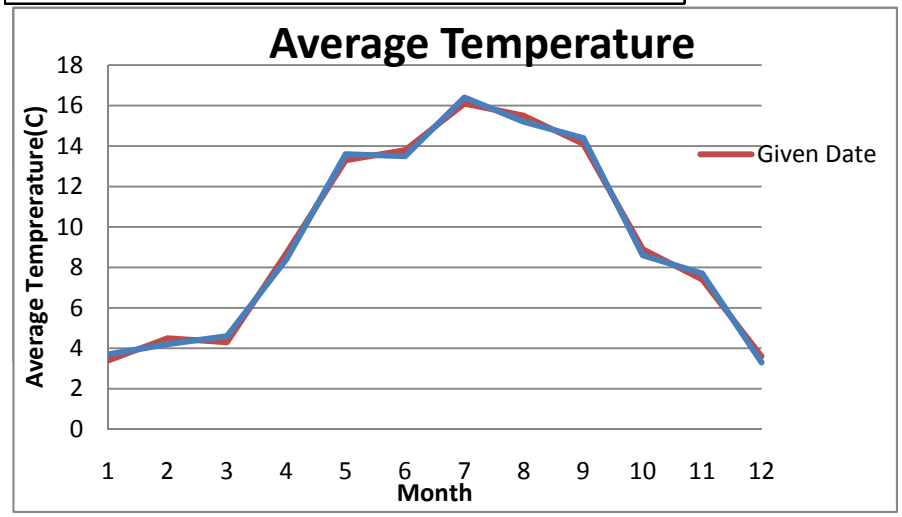
$$x(n) = a_0 + \sum_{k=1}^{n/2} a_k \cos(2\pi k(j-1)/n) + \sum_{k=1}^{n/2-1} b_k \sin(2\pi k(j-1)/n)$$

using follow equations and I got fig(1)

$$a_0 = \frac{1}{n}\sum_{j=1}^{n} x_j$$

$$a_k = \frac{2}{n}\sum_{j=1}^{n} x_j \cos(2\pi(j-1)k/n)$$

$$b_k = \frac{2}{n}\sum_{j=1}^{n} x_j \sin(2\pi(j-1)k/n)$$



figure(1)

n=月	x(n)=平均気温	2π*(n-1)/12
1	3.4	0
2	4.5	0.523598776
3	4.3	1.047197551
4	8.7	1.570796327
5	13.3	2.094395102
6	13.8	2.617993878
7	16.1	3.141592654
8	15.5	3.665191429
9	14.1	4.188790205
10	8.9	4.71238898
11	7.4	5.235987756
12	3.6	5.759586532

Given Date

a0 9.466667

k	ak	bk
1	-6.484956427	-0.66291651
2	0.175	0.04330127
3	0.5	-0.1
4	-0.191666667	0.70725408
5	-0.365043573	0.462916512
6	0.6	1.77634E-15

k	1	2	3	4	5	6	
余弦係数ak	-6.484956427	0.175	0.5	-0.19167	-0.36504	0.6	
正弦係数bk	-0.662916512	0.04330127	-0.1	0.707254	0.462917	0	
n							Fourie series
1	-6.484956427	0.175	0.5	-0.19167	-0.36504	0.6	3.7
2	-5.947595264	0.125	-0.1	0.708333	0.547595	-0.6	4.2
3	-3.816580754	-0.05	-0.5	-0.51667	-0.58342	0.6	4.6
4	-0.662916512	-0.175	0.1	-0.19167	0.462917	-0.6	8.4
5	2.668375673	-0.125	0.5	0.708333	-0.21838	0.6	13.6
6	5.284678752	0.05	-0.1	-0.51667	-0.08468	-0.6	13.5
7	6.484956427	0.175	-0.5	-0.19167	0.365044	0.6	16.4
8	5.947595264	0.125	0.1	0.708333	-0.5476	-0.6	15.2
9	3.816580754	-0.05	0.5	-0.51667	0.583419	0.6	14.4
10	0.662916512	-0.175	-0.1	-0.19167	-0.46292	-0.6	8.6
11	-2.668375673	-0.125	-0.5	0.708333	0.218376	0.6	7.7
12	-5.284678752	0.05	0.1	-0.51667	0.084679	-0.6	3.3

Assignment

28J10025 Risa Kitamoto

The data of the average temperature is given as;

TABLE1 Given data

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average temperature	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

Fourier series is written as;

$$x(n) = a_0 + \sum_{k=1}^{6} a_k cos\,(2\pi f_k n) + \sum_{k=1}^{6} b_k sin\,(2\pi f_k n)$$

The coefficients (a0,a1,a2,$\cdots$,b1,b2,$\cdots$) can be obtained from the following equations;

$$\sum_{n=1}^{12} x(n) = \sum_{n=1}^{12} a_0 + \sum_{n=1}^{12}\sum_{k=1}^{6} a_k cos\,(2\pi f_k n) + \sum_{n=1}^{12}\sum_{k=1}^{6} b_k sin\,(2\pi f_k n)$$

$$\rightarrow a_0 = \frac{1}{N}\sum_{n=1}^{12} x(n)$$

$$\sum_{n=1}^{12} x(n)\,cos(2\pi f_k n) = \sum_{n=1}^{12} a_0 cos(2\pi f_k n) + \sum_{n=1}^{12}\sum_{k=1}^{6} a_k cos^{2}(2\pi f_k n) + \sum_{n=1}^{12}\sum_{k=1}^{6} b_k cos(2\pi f_k n) sin\,(2\pi f_k n)$$

$$\rightarrow a_k = \frac{2}{N}\sum_{n=1}^{12} x(n)\,cos(2\pi f_k n)$$

$$\sum_{n=1}^{12} x(n)\,sin(2\pi f_k n) = \sum_{n=1}^{12} a_0 sin(2\pi f_k n) + \sum_{n=1}^{12}\sum_{k=1}^{6} a_k sin(2\pi f_k n) cos(2\pi f_k n) + \sum_{n=1}^{12}\sum_{k=1}^{6} b_k sin^{2}\,(2\pi f_k n)$$

$$\rightarrow b_k = \frac{2}{N}\sum_{n=1}^{12} x(n)\,sin\,(2\pi f_k n)$$

Here N = 12, $f_k = k/12$

The calculation result of the coefficients is represented in the following table.

TABLE2 Calculation result

k	0	1	2	3	4	5	6
a_k	9.467	−5.28	0.05	0.1	−0.52	0.085	−0.6
b_k		−3.82	0.173	0.5	−0.52	−0.58	5E−15

And the calculation result of average temperature from TABLE2 is represented in the following table and figure.

TABLE3 Calculation result (from TABLE2)

n	1	2	3	4	5	6	7	8	9	10	11	12
$x(n)$	3.7	4.2	4.6	8.4	13.6	13.5	16.4	15.2	14.4	8.6	7.7	3.3

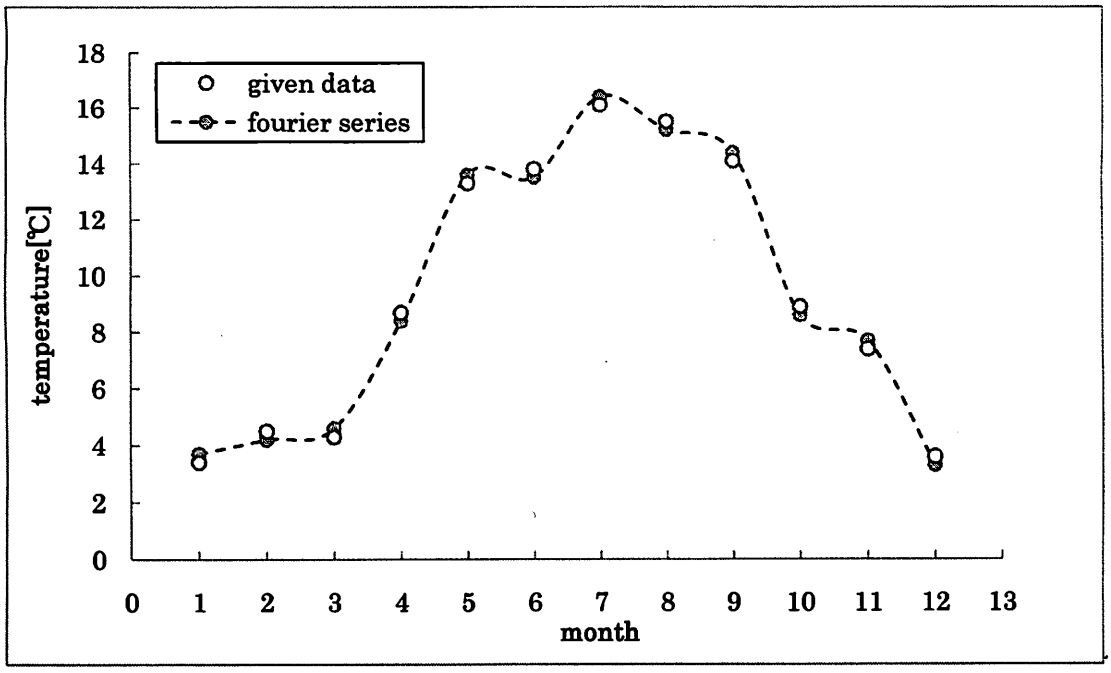


FIGURE1 Average temperature

The given data is represented in the following table.

TABLE3 Given data (from Assignment)

k	1	2	3	4	5	6
a_k	−5.30	0.05	0.10	−0.52	0.09	−0.36
b_k	−3.82	0.17	0.50	−0.52	−0.58	−

I couldn't obtain a6 corresponding with the given data. So I decided to examine the periodicity. I reviewed the coefficient a_k and b_k.

$$\sum_{n=1}^{12} x(n)\cos(2\pi f_k n) = \sum_{n=1}^{12}\sum_{k=1}^{6} a_k \cos^2(2\pi f_k n) \rightarrow \sum_{n=1}^{12} x(n)\cos(2\pi f_k n) = \sum_{n=1}^{12}\sum_{k=1}^{6} a_k \frac{1+\cos(4\pi f_k n)}{2}$$

$$\sum_{n=1}^{12} x(n)\sin(2\pi f_k n) = \sum_{n=1}^{12}\sum_{k=1}^{6} b_k \sin^2(2\pi f_k n) \rightarrow \sum_{n=1}^{12} x(n)\sin(2\pi f_k n) = \sum_{n=1}^{12}\sum_{k=1}^{6} b_k \frac{1-\cos(4\pi f_k n)}{2}$$

I examined the periodicity of $\cos(4\pi f_k n)$. The result is represented in the following table.

TABLE4 $\cos(4\pi f_k n)$

n	1	2	3	4	5	6	7	8	9	10	11	12
$\cos(4\pi f_1 n)$	0.5	−0.5	−1	−0.5	0.5	1	0.5	−0.5	−1	−0.5	0.5	1
$\cos(4\pi f_2 n)$	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1
$\cos(4\pi f_3 n)$	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1
$\cos(4\pi f_4 n)$	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1
$\cos(4\pi f_5 n)$	0.5	−0.5	−1	−0.5	0.5	1	0.5	−0.5	−1	−0.5	0.5	1
$\cos(4\pi f_6 n)$	1	1	1	1	1	1	1	1	1	1	1	1

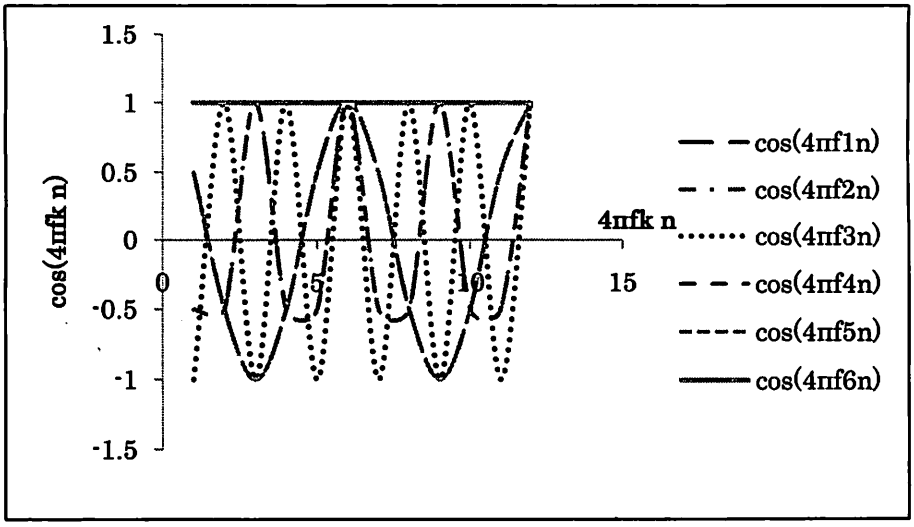


FIGURE2 $cos(4\pi f_k n)$

From the results, I found that I made a mistake. I had to calculate a_6 and b_6 as following equations.

$$a_6 = \frac{1}{N}\sum_{n=1}^{12} x(n)\, cos(2\pi f_k n)$$

$$b_6 = 0$$

The result of recalculation is represented in the following table.

TABLE5 Calculation result

k	0	1	2	3	4	5	6
a_k	9.467	−5.28	0.05	0.1	−0.52	0.085	−0.3
b_k		−3.82	0.173	0.5	−0.52	−0.58	0

And the calculation result of average temperature from TABLE6 is represented in the following table and figure.

TABLE6 Average temperature (new)

n	1	2	3	4	5	6	7	8	9	10	11	12
$x(n)$	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

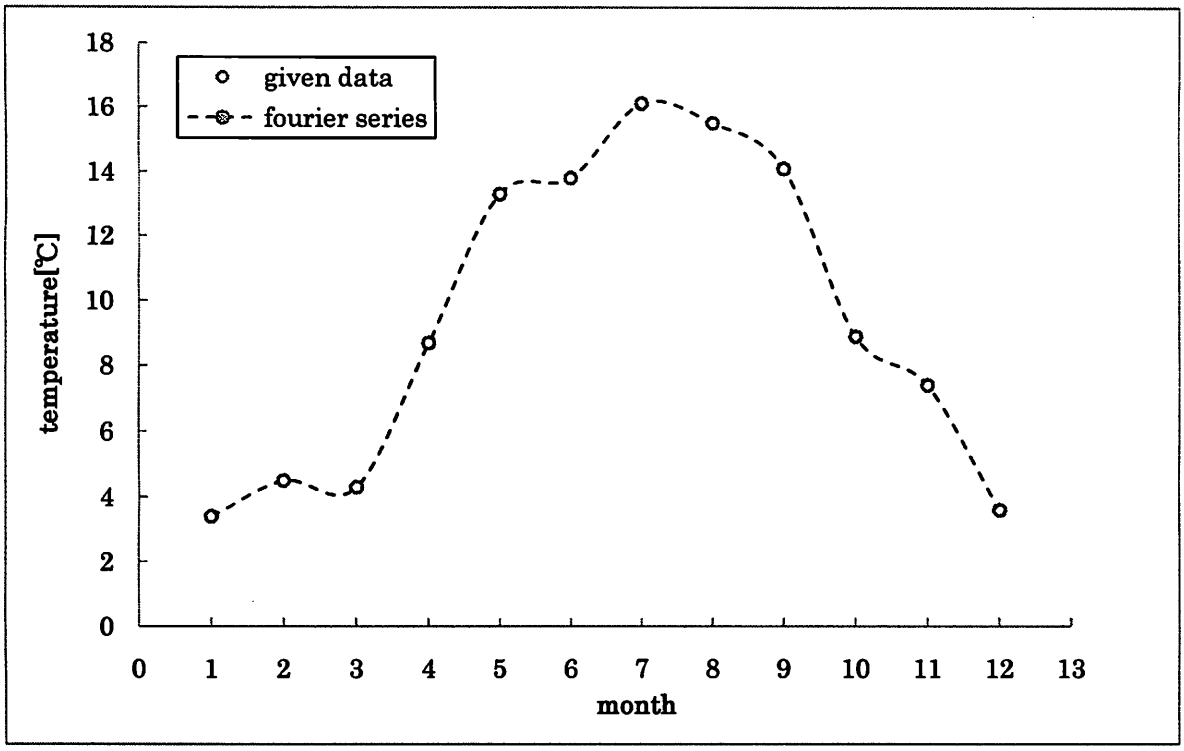


FIGURE3 Average temperature (new)

It is said that the result corresponds with the given data. But a_6 hasn't corresponded with the given data yet. To be honest, I don't know the point.

Consideration

From the table3, it is said that the coefficient a0 means the average value. When I was an undergraduate student, I studied Fourier series. But, I wasn't aware that a0 is the average value. And I know that other coefficients (a1,a2,$\cdots$,b1,b2,$\cdots$) mean the changes from a0(average value). From the viewpoint, I thought that Fourier series had the property similar to Taylor series.

Supplements

Coefficients are written in detail as follows;

$$a_0 = \frac{1}{N}\sum_{n=1}^{12} x(n)$$

$$a_1 = \frac{2}{N}\sum_{n=1}^{12} x(n)\cos(2\pi f_1 n)$$

$$a_2 = \frac{2}{N}\sum_{n=1}^{12} x(n)\cos(2\pi f_2 n)$$

$$\vdots$$

$$a_6 = \frac{1}{N}\sum_{n=1}^{12} x(n)\cos(2\pi f_k n)$$

$$b_1 = \frac{2}{N}\sum_{n=1}^{12} x(n)\sin(2\pi f_1 n)$$

$$b_2 = \frac{2}{N}\sum_{n=1}^{12} x(n)\sin(2\pi f_2 n)$$

$$\vdots$$

$$b_6 = 0$$

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月(n)	平均気温x(r	x(n)withフーリエ
1	3.4	3.4
2	4.5	4.5
3	4.3	4.3
4	8.7	8.7
5	13.3	13.3
6	13.8	13.8
7	16.1	16.1
8	15.5	15.5
9	14.1	14.1
10	8.9	8.9
11	7.4	7.4
12	3.6	3.6

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

Δ t　　1

k	余弦係数ak	正弦係数bk	fk
0	9.46666667		0
1	−5.2846788	−3.8165808	0.083333
2	0.05	0.17320508	0.166667
3	0.1	0.5	0.25
4	−0.5166667	−0.5196152	0.333333
5	0.08467875	−0.5834192	0.416667
6	−0.3		0.5

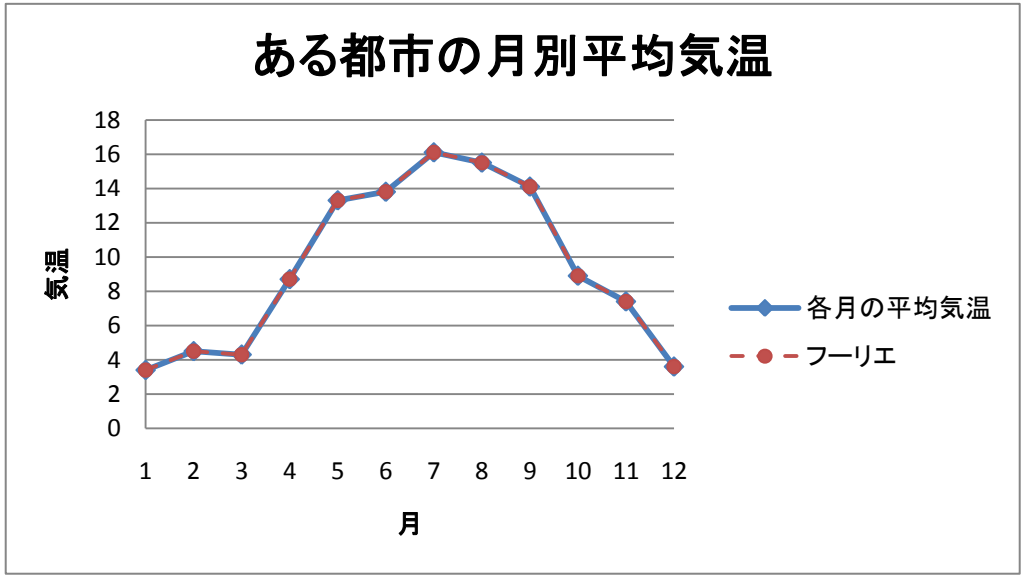
$$a_0 = \frac{1}{12}\sum_{n=1}^{12} x(n)$$

$$a_k = 2 \cdot \frac{1}{12}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n$$

when　k=6

$$a_6 = \frac{1}{12}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n$$

$$b_k = 2 \cdot \frac{1}{12}\sum_{n=1}^{12} x(n)\sin 2\pi f_k n$$



$\cos 2\pi f_k n$

k/n	1	2	3	4	5	6	7	8	9	10	11	12
1	0.866025404	0.5	6.12574E-17	−0.5	−0.86603	−1	−0.8660254	−0.5	−1.8E-16	0.5	0.866025	1
2	0.5	−0.5	−1	−0.5	0.5	1	0.5	−0.5	−1	−0.5	0.5	1
3	6.12574E-17	−1	−1.83772E-16	1	3.06E-16	−1	−4.288E-16	1	5.51E-16	−1	−2.5E-15	1
4	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1	−0.5	−0.5	1
5	−0.8660254	0.5	3.06287E-16	−0.5	0.866025	−1	0.8660254	−0.5	8.57E-16	0.5	−0.86603	1
6	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1

$\sin 2\pi f_k n$

k/n	1	2	3	4	5	6	7	8	9	10	11	12
1	0.5	0.8660254	1	0.866025	0.5	1.2251E-16	−0.5	−0.86603	−1	−0.86603	−0.5	−2.5E-16
2	0.866025404	0.8660254	1.22515E-16	−0.86603	−0.86603	−2.45E-16	0.8660254	0.866025	3.68E-16	−0.86603	−0.86603	−4.9E-16
3	1	1.225E-16	−1	−2.5E-16	1	3.6754E-16	−1	−4.9E-16	1	6.13E-16	−1	−7.4E-16
4	0.866025404	−0.866025	−2.4503E-16	0.866025	−0.86603	−4.901E-16	0.8660254	−0.86603	−7.4E-16	0.866025	−0.86603	−9.8E-16
5	0.5	−0.866025	1	−0.86603	0.5	6.1257E-16	−0.5	0.866025	−1	0.866025	−0.5	−1.2E-15

$x(n)\cos 2\pi f_k n$

k/x(n)	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6
1	2.944486373	2.25	2.63407E-16	−4.35	−11.5181	−13.8	−13.943009	−7.75	−2.6E-15	4.45	6.408588	3.6
2	1.7	−2.25	−4.3	−4.35	6.65	13.8	8.05	−7.75	−14.1	−4.45	3.7	3.6
3	2.08275E-16	−4.5	−7.90221E-16	8.7	4.07E-15	−13.8	−6.904E-15	15.5	7.77E-15	−8.9	−1.8E-14	3.6
4	−1.7	−2.25	4.3	−4.35	−6.65	13.8	−8.05	−7.75	14.1	−4.45	−3.7	3.6
5	−2.94448637	2.25	1.31703E-15	−4.35	11.51814	−13.8	13.943009	−7.75	1.21E-14	4.45	−6.40859	3.6
6	−3.4	4.5	−4.3	8.7	−13.3	13.8	−16.1	15.5	−14.1	8.9	−7.4	3.6

$x(n)\sin 2\pi f_k n$

k/x(n)	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6
1	1.7	3.8971143	4.3	7.534421	6.65	1.6907E-15	−8.05	−13.4234	−14.1	−7.70763	−3.7	−8.8E-16
2	2.944486373	3.8971143	5.26814E-16	−7.53442	−11.5181	−3.381E-15	13.943009	13.42339	5.18E-15	−7.70763	−6.40859	−1.8E-15
3	3.4	5.513E-16	−4.3	−2.1E-15	13.3	5.0721E-15	−16.1	−7.6E-15	14.1	5.45E-15	−7.4	−2.6E-15
4	2.944486373	−3.897114	−1.05363E-15	7.534421	−11.5181	−6.763E-15	13.943009	−13.4234	−1E-14	7.707626	−6.40859	−3.5E-15
5	1.7	−3.897114	4.3	−7.53442	6.65	8.4535E-15	−8.05	13.42339	−14.1	7.707626	−3.7	−4.4E-15

$a_k \cos 2\pi f_k n$

k/n	1	2	3	4	5	6	7	8	9	10	11	12
1	−4.57666605	−2.642339	−3.23726E−16	2.642339	4.576666	5.28467875	4.57666605	2.642339	9.71E−16	−2.64234	−4.57667	−5.28468
2	0.025	−0.025	−0.05	−0.025	0.025	0.05	0.025	−0.025	−0.05	−0.025	0.025	0.05
3	6.12574E−18	−0.1	−1.83772E−17	0.1	3.06E−17	−0.1	−4.288E−17	0.1	5.51E−17	−0.1	−2.5E−16	0.1
4	0.258333333	0.2583333	−0.516666667	0.258333	0.258333	−0.5166667	0.25833333	0.258333	−0.51667	0.258333	0.258333	−0.51667
5	−0.07333395	0.0423394	2.5936E−17	−0.04234	0.073334	−0.0846788	0.07333395	−0.04234	7.26E−17	0.042339	−0.07333	0.084679
6	0.3	−0.3	0.3	−0.3	0.3	−0.3	0.3	−0.3	0.3	−0.3	0.3	−0.3

$b_k \sin 2\pi f_k n$

k/n	1	2	3	4	5	6	7	8	9	10	11	12
1	−1.90829038	−3.305256	−3.816580754	−3.30526	−1.90829	−4.676E−16	1.90829038	3.305256	3.816581	3.305256	1.90829	9.35E−16
2	0.15	0.15	2.12202E−17	−0.15	−0.15	−4.244E−17	0.15	0.15	6.37E−17	−0.15	−0.15	−8.5E−17
3	0.5	6.126E−17	−0.5	−1.2E−16	0.5	1.8377E−16	−0.5	−2.5E−16	0.5	3.06E−16	−0.5	−3.7E−16
4	−0.45	0.45	1.27321E−16	−0.45	0.45	2.5464E−16	−0.45	0.45	3.82E−16	−0.45	0.45	5.09E−16
5	−0.29170962	0.5052559	−0.583419246	0.505256	−0.29171	−3.574E−16	0.29170962	−0.50526	0.583419	−0.50526	0.29171	7.15E−16

少数の離散的な時系列を使ってフーリエ級数について係数の計算例を示す。下表はある都市の月別平均気温である。

月	1	2	3	4	5	6	7	8	9	10	11	12
平均気温	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

この $N = 12$ 個の等間隔に与えられた離散時系列 $\{x(n), n = 1, 2, \cdots, 12\}$ に対して、正弦関数と余弦関数の振幅を適切に調整することによってそれらの和

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

が元の時系列の各点を通るようにすることを考える（$\Delta t = 1$）。ここで、$f_k = k/12$ は周波数である。上の時系列の場合、データ個数 N は 12 個であるから偶数の場合に当たり、式 (1.16)、(1.17) 等を使って計算すると下表のようになる。

k	余弦係数 a_k	正弦係数 b_k
1	−5.30	−3.82
2	0.05	0.17
3	0.10	0.50
4	−0.52	−0.52
5	0.09	−0.58
6	−0.36	-

・計算結果

	表による係数		計算した係数	
k	ak	bk	ak	bk
0.00			9.47	
1.00	−5.30	−3.82	−5.28	−3.82
2.00	0.05	0.17	0.05	0.17
3.00	0.10	0.50	0.10	0.50
4.00	−0.52	−0.52	−0.52	−0.52
5.00	0.09	−0.58	0.08	−0.58
6.00	−0.36		−0.30	

Δt=	1

月	平均気温	離散フーリエ変換結
1	3.4	3.4
2	4.5	4.5
3	4.3	4.3
4	8.7	8.7
5	13.3	13.3
6	13.8	13.8
7	16.1	16.1
8	15.5	15.5
9	14.1	14.1
10	8.9	8.9
11	7.4	7.4
12	3.6	3.6

・係数を求める

		k					
月	平均気温	1	2	3	4	5	6
1	3.4	2.94	1.70	0.00	−1.70	−2.94	−3.40
2	4.5	2.25	−2.25	−4.50	−2.25	2.25	4.50
3	4.3	0.00	−4.30	0.00	4.30	0.00	−4.30
4	8.7	−4.35	−4.35	8.70	−4.35	−4.35	8.70
5	13.3	−11.52	6.65	0.00	−6.65	11.52	−13.30
6	13.8	−13.80	13.80	−13.80	13.80	−13.80	13.80
7	16.1	−13.94	8.05	0.00	−8.05	13.94	−16.10
8	15.5	−7.75	−7.75	15.50	−7.75	−7.75	15.50
9	14.1	0.00	−14.10	0.00	14.10	0.00	−14.10
10	8.9	4.45	−4.45	−8.90	−4.45	4.45	8.90
11	7.4	6.41	3.70	0.00	−3.70	−6.41	−7.40
12	3.6	3.60	3.60	3.60	3.60	3.60	3.60
	sum	−31.71	0.30	0.60	−3.10	0.51	−3.60
	ak	−5.28	0.05	0.10	−0.52	0.08	−0.30

		k				
月	平均気温	1	2	3	4	5
	3.4	1.70	2.94	3.40	2.94	1.70
2	4.5	3.90	3.90	0.00	−3.90	−3.90
3	4.3	4.30	0.00	−4.30	0.00	4.30
4	8.7	7.53	−7.53	0.00	7.53	−7.53
5	13.3	6.65	−11.52	13.30	−11.52	6.65
6	13.8	0.00	0.00	0.00	0.00	0.00
7	16.1	−8.05	13.94	−16.10	13.94	−8.05
8	15.5	−13.42	13.42	0.00	−13.42	13.42
9	14.1	−14.10	0.00	14.10	0.00	−14.10
10	8.9	−7.71	−7.71	0.00	7.71	7.71
11	7.4	−3.70	−6.41	−7.40	−6.41	−3.70
12	3.6	0.00	0.00	0.00	0.00	0.00
	sum	−22.90	1.04	3.00	−3.12	−3.50
	bk	−3.82	0.17	0.50	−0.52	−0.58

$$a_0 = \frac{1}{12}\sum_{k=1}^{12} x(k)\Delta t$$

$$a_k = \frac{2}{12}\sum_{n=1}^{12} x(n)\{\cos(2\pi f_k n)\}\Delta t$$

except for

$$a_6 = \frac{1}{12}\sum_{n=1}^{12} x(n)\{\cos(2\pi f_k n)\}\Delta t$$

$$b_k = \frac{2}{12}\sum_{n=1}^{12} x(n)\{\sin(2\pi f_k n)\}\Delta t$$

COS成分

n=1	3.4	n=2	4.5	n=3	4.3	n=4	8.7	n=5	13.3	n=6	13.8	n=7	16.1	n=8	15.5	n=9	14.1	n=10	8.9	n=11	7.4	n=12	3.6
1		2		3		4		5		6		7		8		9		10		11		12	
1	0.866	1	0.5	1	6E−17	1	−0.5	1	−0.866	1	−1	1	−0.866	1	−0.5	1	−2E−16	1	0.5	1	0.866	1	1
2	0.5	2	−0.5	2	−1	2	−0.5	2	0.5	2	1	2	0.5	2	−0.5	2	−1	2	−0.5	2	0.5	2	1
3	6E−17	3	−1	3	−2E−16	3	1	3	3E−16	3	−1	3	−4E−16	3	1	3	5.5E−16	3	−1	3	−2E−15	3	1
4	−0.5	4	−0.5	4	1	4	−0.5	4	−0.5	4	1	4	−0.5	4	−0.5	4	1	4	−0.5	4	−0.5	4	1
5	−0.866	5	0.5	5	3E−16	5	−0.5	5	0.866	5	−1	5	0.866	5	−0.5	5	8.6E−16	5	0.5	5	−0.866	5	1
6	−1	6	1	6	−1	6	1	6	−1	6	1	6	−1	6	1	6	−1	6	1	6	−1	6	1

SIN成分

1		2		3		4		5		6		7		8		9		10		11		12	
1	0.5	1	0.86603	1	1	1	0.866	1	0.5	1	1E−16	1	−0.5	1	−0.866	1	−1	1	−0.866	1	−0.5	1	−2.5E−16
2	0.866	2	0.86603	2	1E−16	2	−0.866	2	−0.866	2	−2E−16	2	0.866	2	0.86603	2	3.7E−16	2	−0.866	2	−0.866	2	−4.9E−16
3	1	3	1.2E−16	3	−1	3	−2E−16	3	1	3	4E−16	3	−1	3	−5E−16	3	1	3	6E−16	3	−1	3	−7.4E−16
4	0.866	4	−0.866	4	−2E−16	4	0.866	4	−0.866	4	−5E−16	4	0.866	4	−0.866	4	−7E−16	4	0.866	4	−0.866	4	−9.8E−16
5	0.5	5	−0.866	5	1	5	−0.866	5	0.5	5	6E−16	5	−0.5	5	0.86603	5	−1	5	0.866	5	−0.5	5	−1.2E−15

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次の離散的な時系列を使ってフーリエ級数について係数の計算例を示す。下表はある都市の月別平均気温である。

月	1	2	3	4	5	6	7	8	9	10	11	12
平均気温	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

この $N = 12$ 個の等間隔に与えられた離散時系列 $\{x(n), n = 1, 2, \cdots, 12\}$ に対して、正弦関数と余弦関数の振幅を適切に調整することによってそれらの和

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

が元の時系列の各点を通るようにすることを考える（$\Delta t = 1$）。ここで、$f_k = k/12$ は周波数である。上の時系列の場合、データ個数 N は 12 個であるから偶数の場合に当たり、式 (1.16)、(1.17) 等を使って計算すると下表のようになる。

k	余弦係数 a_k	正弦係数 b_k
1	−5.30	−3.82
2	0.05	0.17
3	0.10	0.50
4	−0.52	−0.52
5	0.09	−0.58
6	−0.36	-

計算結果（表2より）

k	ak	bk
0	9.4667	
1	−5.285	−3.817
2	0.05	0.1732
3	0.1	0.5
4	−0.517	−0.52
5	0.0847	−0.583
6	−0.3	

解答
上記のフーリエ級数にfk,nの値を代入し、12元連立方程式を立てる（表1）。

表1

		k= 0	k= 1	k= 2	k= 3	k= 4	k= 5	k= 6	k= 1	k= 2	k= 3	k= 4	k= 5		x(n)
n=	1	1	0.866	0.5	6E−17	−0.5	−0.866	−1	0.5	0.866	1	0.866	0.5	a0	3.4
n=	2	1	0.5	−0.5	−1	−0.5	0.5	1	0.866	0.866	1E−16	−0.866	−0.866	a1	4.5
n=	3	1	6E−17	−1	−2E−16	1	3E−16	−1	1	1E−16	−1	−2E−16	1	a2	4.3
n=	4	1	−0.5	−0.5	1	−0.5	−0.5	1	0.866	−0.866	−2E−16	0.866	−0.866	a3	8.7
n=	5	1	−0.866	0.5	3E−16	−0.5	0.866	−1	0.5	−0.866	1	−0.866	0.5	a4	13.3
n=	6	1	−1	1	−1	1	−1	1	1E−16	−2E−16	4E−16	−5E−16	6E−16	a5	13.8
n=	7	1	−0.866	0.5	−4E−16	−0.5	0.866	−1	−0.5	0.866	−1	0.866	−0.5	a6	16.1
n=	8	1	−0.5	−0.5	1	−0.5	−0.5	1	−0.866	0.866	−5E−16	−0.866	0.866	b1	15.5
n=	9	1	−2E−16	−1	6E−16	1	9E−16	−1	−1	4E−16	1	−7E−16	−1	b2	14.1
n=	10	1	0.5	−0.5	−1	−0.5	0.5	1	−0.866	−0.866	6E−16	0.866	0.866	b3	8.9
n=	11	1	0.866	0.5	−2E−15	−0.5	−0.866	−1	−0.5	−0.866	−1	−0.866	−0.5	b4	7.4
n=	12	1	1	1	1	1	1	1	−2E−16	−5E−16	−7E−16	−1E−15	−1E−15	b5	3.6

表1の連立方程式の係数行列の逆行列を計算し、解a0～a11を求める(表2)。

表2

													計算結果	確認
0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	9.4666667	3.4
0.1443	0.0833	4E-17	-0.083	-0.144	-0.167	-0.144	-0.083	-2E-16	0.0833	0.1443	0.1667	-5.284679	4.5
0.0833	-0.083	-0.167	-0.083	0.0833	0.1667	0.0833	-0.083	-0.167	-0.083	0.0833	0.1667	0.05	4.3
2E-16	-0.167	1E-16	0.1667	-5E-17	-0.167	-2E-16	0.1667	1E-16	-0.167	-2E-16	0.1667	0.1	8.7
-0.083	-0.083	0.1667	-0.083	-0.083	0.1667	-0.083	-0.083	0.1667	-0.083	-0.083	0.1667	-0.516667	13.3
-0.144	0.0833	-5E-17	-0.083	0.1443	-0.167	0.1443	-0.083	-3E-17	0.0833	-0.144	0.1667	0.0846788	13.8
-0.083	0.0833	-0.083	0.0833	-0.083	0.0833	-0.083	0.0833	-0.083	0.0833	-0.083	0.0833	-0.3	16.1
0.0833	0.1443	0.1667	0.1443	0.0833	1E-16	-0.083	-0.144	-0.167	-0.144	-0.083	-7E-17	-3.816581	15.5
0.1443	0.1443	2E-16	-0.144	-0.144	-5E-17	0.1443	0.1443	2E-16	-0.144	-0.144	-1E-16	0.1732051	14.1
0.1667	1E-16	-0.167	-2E-17	0.1667	1E-16	-0.167	-2E-16	0.1667	7E-17	-0.167	-9E-17	0.5	8.9
0.1443	-0.144	-3E-16	0.1443	-0.144	-2E-16	0.1443	-0.144	-2E-16	0.1443	-0.144	-2E-16	-0.519615	7.4
0.0833	-0.144	0.1667	-0.144	0.0833	-6E-17	-0.083	0.1443	-0.167	0.1443	-0.083	7E-17	-0.583419	3.6

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b=A × x(n)

A=

1	COS(PI()/6)	cos(pi()/3)	cos(pi()/2)	cos(2*pi()/3)	cos(5*pi()/6)
1	cos(2*pi()/6)	cos(2*pi()/3)	cos(2*pi()/2)	cos(2*2*pi()/3)	cos(5*2*pi()/6)
1	cos(3*pi()/6)	cos(3*pi()/3)	cos(3*pi()/2)	cos(2*3*pi()/3)	cos(5*3*pi()/6)
1	cos(4*pi()/6)	cos(4*pi()/3)	cos(4*pi()/2)	cos(2*4*pi()/3)	cos(5*4*pi()/6)
1	cos(5*pi()/6)	cos(5*pi()/3)	cos(5*pi()/2)	cos(2*5*pi()/3)	cos(5*5*pi()/6)
1	cos(6*pi()/6)	cos(6*pi()/3)	cos(6*pi()/2)	cos(2*6*pi()/3)	cos(5*6*pi()/6)
1	cos(7*pi()/6)	cos(7*pi()/3)	cos(7*pi()/2)	cos(2*7*pi()/3)	cos(5*7*pi()/6)
1	cos(8*pi()/6)	cos(8*pi()/3)	cos(8*pi()/2)	cos(2*8*pi()/3)	cos(5*8*pi()/6)
1	cos(9*pi()/6)	cos(9*pi()/3)	cos(9*pi()/2)	cos(2*9*pi()/3)	cos(5*9*pi()/6)
1	cos(10*pi()/6)	cos(10*pi()/3)	cos(10*pi()/2)	cos(2*10*pi()/3)	cos(5*10*pi()/6)
1	cos(11*pi()/6)	cos(11*pi()/3)	cos(11*pi()/2)	cos(2*11*pi()/3)	cos(5*11*pi()/6)
1	cos(12*pi()/6)	cos(12*pi()/3)	cos(12*pi()/2)	cos(2*12*pi()/3)	cos(5*12*pi()/6)

-(1)

1	0.866025404	0.5	6.12574E-17	-0.5	-0.866025404
1	0.5	-0.5	-1	-0.5	0.5
1	6.12574E-17	-1	-1.83772E-16	1	1.19447E-15
1	-0.5	-0.5	1	-0.5	-0.5
1	-0.866025404	0.5	3.06287E-16	-0.5	0.866025404
1	-1	1	-1	1	-1
1	-0.866025404	0.5	-4.28802E-16	-0.5	0.866025404
1	-0.5	-0.5	1	-0.5	-0.5
1	-1.83772E-16	-1	5.51317E-16	1	8.57495E-16
1	0.5	-0.5	-1	-0.5	0.5
1	0.866025404	0.5	-2.45019E-15	-0.5	-0.866025404
1	1	1	1	1	1

-(2)

A^{-1}=

0.08333333	0.083333333	0.083333333	0.083333333	0.083333333	0.083333333
0.14433757	0.083333333	2.00801E-16	-0.083333333	-0.144337567	-0.166666667
0.08333333	-0.083333333	-0.166666667	-0.083333333	0.083333333	0.166666667
1.4985E-16	-0.166666667	1.72707E-16	0.166666667	2.04455E-16	-0.166666667
-0.0833333	-0.083333333	0.166666667	-0.083333333	-0.083333333	0.166666667
-0.1443376	0.083333333	1.26383E-16	-0.083333333	0.144337567	-0.166666667
-0.0833333	0.083333333	-0.083333333	0.083333333	-0.083333333	0.083333333
0.08333333	0.144337567	0.166666667	0.144337567	0.083333333	5.61054E-17
0.14433757	0.144337567	-4.27325E-17	-0.144337567	-0.144337567	-3.07244E-17
0.16666667	1.50961E-16	-0.166666667	7.21111E-17	0.166666667	1.88772E-17
0.14433757	-0.144337567	-1.60247E-16	0.144337567	-0.144337567	-1.20863E-16
0.08333333	-0.144337567	0.166666667	-0.144337567	0.083333333	-2.33504E-17

-(3)

x(n)=

	n
3.4	1
4.5	2
4.3	3
8.7	4
13.3	5
13.8	6
16.1	7
15.5	8
14.1	9
8.9	10
7.4	11
3.6	12

a0=	9.466666667
a1=	-5.284678752
a2=	0.05
a3=	0.1
a4=	-0.516666667
a5=	0.084678752
a6=	-0.3
b1=	-3.816580754
b2=	0.173205081
b3=	0.5
b4=	-0.519615242
b5=	-0.583419246

①

cos(pi())	sin(pi()/6)	sin(pi()/3)	sin(pi()/2)	sin(2*pi()/3)	sin(5*pi()/6)
cos(2*pi())	sin(2*pi()/6)	sin(2*pi()/3)	sin(2*pi()/2)	sin(2*2*pi()/3)	sin(5*2*pi()/6)
cos(3*pi())	sin(3*pi()/6)	sin(3*pi()/3)	sin(3*pi()/2)	sin(2*3*pi()/3)	sin(5*3*pi()/6)
cos(4*pi())	sin(4*pi()/6)	sin(4*pi()/3)	sin(4*pi()/2)	sin(2*4*pi()/3)	sin(5*4*pi()/6)
cos(5*pi())	sin(5*pi()/6)	sin(5*pi()/3)	sin(5*pi()/2)	sin(2*5*pi()/3)	sin(5*5*pi()/6)
cos(6*pi())	sin(6*pi()/6)	sin(6*pi()/3)	sin(6*pi()/2)	sin(2*6*pi()/3)	sin(5*6*pi()/6)
cos(7*pi())	sin(7*pi()/6)	sin(7*pi()/3)	sin(7*pi()/2)	sin(2*7*pi()/3)	sin(5*7*pi()/6)
cos(8*pi())	sin(8*pi()/6)	sin(8*pi()/3)	sin(8*pi()/2)	sin(2*8*pi()/3)	sin(5*8*pi()/6)
cos(9*pi())	sin(9*pi()/6)	sin(9*pi()/3)	sin(9*pi()/2)	sin(2*9*pi()/3)	sin(5*9*pi()/6)
cos(10*pi())	sin(10*pi()/6)	sin(10*pi()/3)	sin(10*pi()/2)	sin(2*10*pi()/3	sin(5*10*pi()/6)
cos(11*pi())	sin(11*pi()/6)	sin(11*pi()/3)	sin(11*pi()/2)	sin(2*11*pi()/3	sin(5*11*pi()/6)
cos(12*pi())	sin(12*pi()/6)	sin(12*pi()/3)	sin(12*pi()/2)	sin(2*12*pi()/3	sin(5*12*pi()/6)

②

-1	0.5	0.866025404	1	0.866025404	0.5
1	0.866025404	0.866025404	1.22515E-16	-0.866025404	-0.866025404
-1	1	1.22515E-16	-1	-2.4503E-16	1
1	0.866025404	-0.8660254	-2.4503E-16	0.866025404	-0.866025404
-1	0.5	-0.8660254	1	-0.866025404	0.5
1	1.22515E-16	-2.4503E-16	3.67545E-16	-4.90059E-16	2.38893E-15
-1	-0.5	0.866025404	-1	0.866025404	-0.5
1	-0.8660254	0.866025404	-4.9006E-16	-0.866025404	0.866025404
-1	-1	3.67545E-16	1	-7.35089E-16	-1
1	-0.8660254	-0.8660254	6.12574E-16	0.866025404	0.866025404
-1	-0.5	-0.8660254	-1	-0.866025404	-0.5
1	-2.4503E-16	-4.9006E-16	-7.3509E-16	-9.80119E-16	-4.77786E-15

③

0.0833333	0.083333333	0.083333333	0.083333333	0.083333333	0.083333333
-0.144338	-0.08333333	-2.008E-16	0.083333333	0.144337567	0.166666667
0.0833333	-0.08333333	-0.16666667	-0.08333333	0.083333333	0.166666667
-3.23E-16	0.166666667	-1.1175E-16	-0.16666667	-1.04132E-16	0.166666667
-0.083333	-0.08333333	0.166666667	-0.08333333	-0.083333333	0.166666667
0.1443376	-0.08333333	-2.0221E-16	0.083333333	-0.144337567	0.166666667
-0.083333	0.083333333	-0.08333333	0.083333333	-0.083333333	0.083333333
-0.083333	-0.14433757	-0.16666667	-0.14433757	-0.083333333	-1.69881E-16
0.1443376	0.144337567	2.35029E-16	-0.14433757	-0.144337567	-9.78069E-17
-0.166667	-2.0547E-16	0.166666667	1.73601E-16	-0.166666667	-1.36067E-16
0.1443376	-0.14433757	-9.6148E-17	0.144337567	-0.144337567	-1.35198E-16
-0.083333	0.144337567	-0.16666667	0.144337567	-0.083333333	3.84398E-17

28J10096 山本祐樹

observed data

n 月	x(n) 平均気温	2πfk
1	3.4	0.523599
2	4.5	1.047198
3	4.3	1.570796
4	8.7	2.094395
5	13.3	2.617994
6	13.8	3.141593
7	16.1	3.665191
8	15.5	4.18879
9	14.1	4.712389
10	8.9	5.235988
11	7.4	5.759587
12	3.6	6.283185

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

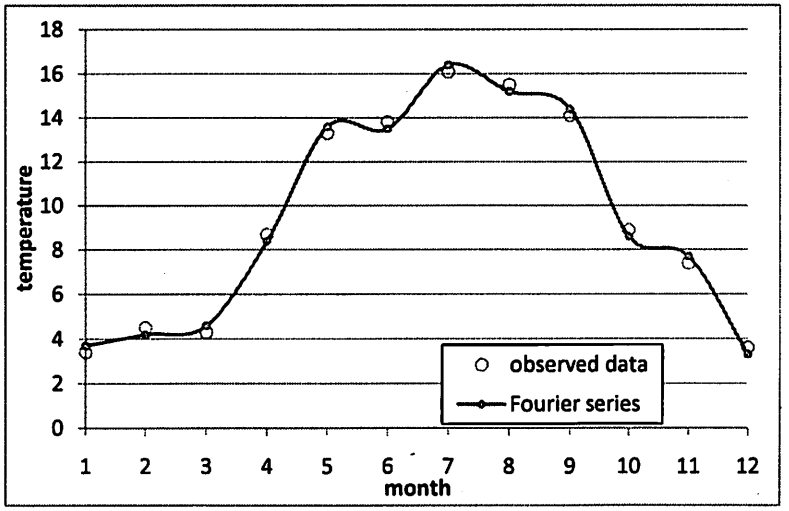
$$a_k = \frac{1}{6}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n, \quad b_k = \frac{1}{6}\sum_{n=1}^{12} x(n)\sin 2\pi f_k n$$

Δt = 1
fk = k/12

k	0	1	2	3	4	5	6
ak	9.466667	−5.28468	0.05	0.1	−0.51667	0.084679	−0.6
bk	0	−3.81658	0.173205	0.5	−0.51962	−0.58342	4.81E−15

Fourier series

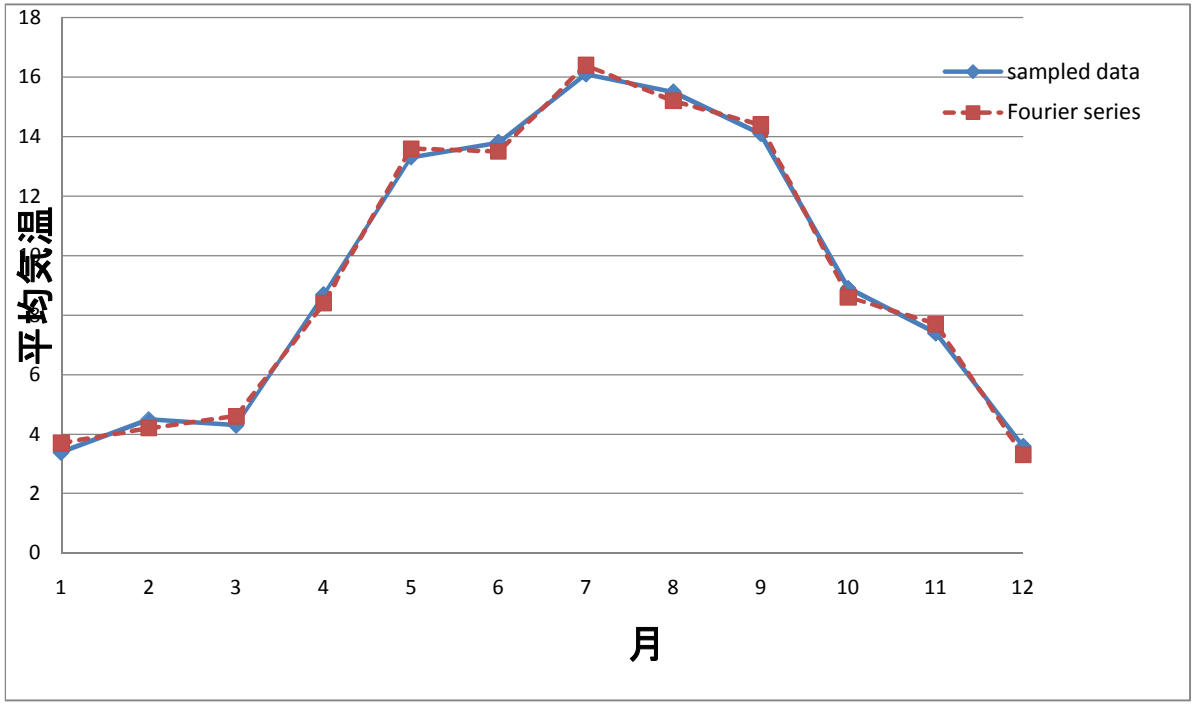
n	k=0	k=1	k=2	k=3	k=4	k=5	k=6	siguma
1	9.466667	−6.48496	0.175	0.5	−0.19167	−0.36504	0.6	3.7
2	9.466667	−5.9476	0.125	−0.1	0.708333	0.547595	−0.6	4.2
3	9.466667	−3.81658	−0.05	−0.5	−0.51667	−0.58342	0.6	4.6
4	9.466667	−0.66292	−0.175	0.1	−0.19167	0.462917	−0.6	8.4
5	9.466667	2.668376	−0.125	0.5	0.708333	−0.21838	0.6	13.6
6	9.466667	5.284679	0.05	−0.1	−0.51667	−0.08468	−0.6	13.5
7	9.466667	6.484956	0.175	−0.5	−0.19167	0.365044	0.6	16.4
8	9.466667	5.947595	0.125	0.1	0.708333	−0.5476	−0.6	15.2
9	9.466667	3.816581	−0.05	0.5	−0.51667	0.583419	0.6	14.4
10	9.466667	0.662917	−0.175	−0.1	−0.19167	−0.46292	−0.6	8.6
11	9.466667	−2.66838	−0.125	−0.5	0.708333	0.218376	0.6	7.7
12	9.466667	−5.28468	0.05	0.1	−0.51667	0.084679	−0.6	3.3



n

月	平均気温	x(n)
1	3.4	3.7
2	4.5	4.2
3	4.3	4.6
4	8.7	8.4
5	13.3	13.6
6	13.8	13.5
7	16.1	16.4
8	15.5	15.2
9	14.1	14.4
10	8.9	8.6
11	7.4	7.7
12	3.6	3.3

k	k/12	a0 9.466667 ak	bk
1	0.083333	−5.28468	−3.81658
2	0.166667	0.05	0.173205
3	0.25	0.1	0.5
4	0.333333	−0.51667	−0.51962
5	0.416667	0.084679	−0.58342
6	0.5	−0.6	



Name: Guangyin Xu
Stu. ID: 28J10101

Month	Average Temperature
n	x(n)
1	3.4
2	4.5
3	4.3
4	8.7
5	13.3
6	13.8
7	16.1
8	15.5
9	14.1
10	8.9
11	7.4
12	3.6

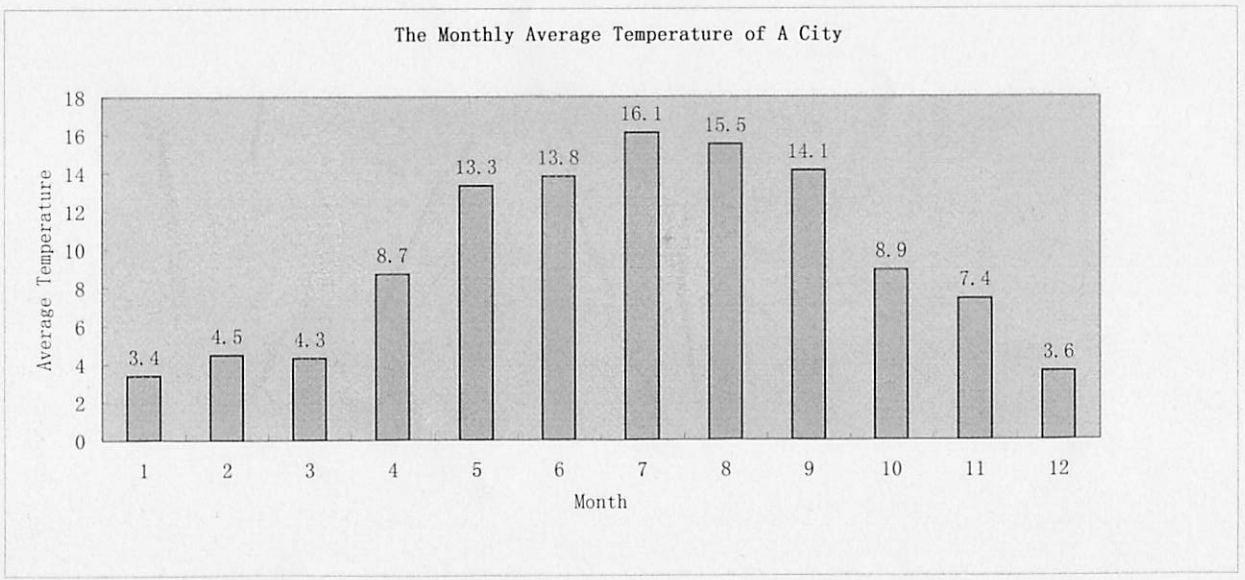
$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

$$a_0 = \frac{1}{12}\sum_{n=1}^{12} x(n)$$

$$a_k = 2 \times \frac{1}{12}\sum_{n=1}^{12} x(n)\cos\ 2\pi f_k n$$

$$b_k = 2 \times \frac{1}{12}\sum_{n=1}^{12} x(n)\sin\ 2\pi f_k n$$

$$a_6 = \frac{1}{12}\sum_{n=1}^{12} x(n)\cos\ 2\pi f_6 n$$



	cos(2πfkn)						sin(2πfkn)				
n \ k	1	2	3	4	5	6	1	2	3	4	5
1	0.866025	0.5	6.13E-17	-0.5	-0.86603	-1	0.5	0.866025	1	0.866025	0.5
2	0.5	-0.5	-1	-0.5	0.5	1	0.866025	0.866025	1.23E-16	-0.86603	-0.86603
3	6.13E-17	-1	-1.8E-16	1	1.19E-15	-1	1	1.23E-16	-1	-2.5E-16	1
4	-0.5	-0.5	1	-0.5	-0.5	1	0.866025	-0.86603	-2.5E-16	0.866025	-0.86603
5	-0.86603	0.5	1.19E-15	-0.5	0.866025	-1	0.5	-0.86603	1	-0.86603	0.5
6	-1	1	-1	1	-1	1	1.23E-16	-2.5E-16	3.68E-16	-4.9E-16	2.39E-15
7	-0.86603	0.5	-4.3E-16	-0.5	0.866025	-1	-0.5	0.866025	-1	0.866025	-0.5
8	-0.5	-0.5	1	-0.5	-0.5	1	-0.86603	0.866025	-4.9E-16	-0.86603	0.866025
9	-1.8E-16	-1	5.51E-16	1	8.57E-16	-1	-1	3.68E-16	1	-7.4E-16	-1
10	0.5	-0.5	-1	-0.5	0.5	1	-0.86603	-0.86603	2.39E-15	0.866025	0.866025
11	0.866025	0.5	1.1E-15	-0.5	-0.86603	-1	-0.5	-0.86603	-1	-0.86603	-0.5
12	1	1	1	1	1	1	-2.5E-16	-4.9E-16	-7.4E-16	-9.8E-16	-4.8E-15

	X(n)*cos(2πfkn)						X(n)*sin(2πfkn)				
n \ k	1	2	3	4	5	6	1	2	3	4	5
1	2.944486	1.7	2.08E-16	-1.7	-2.94449	-3.4	1.7	2.944486	3.4	2.944486	1.7
2	2.25	-2.25	-4.5	-2.25	2.25	4.5	3.897114	3.897114	5.51E-16	-3.89711	-3.89711
3	2.63E-16	-4.3	-7.9E-16	4.3	5.14E-15	-4.3	4.3	5.27E-16	-4.3	-1.1E-15	4.3
4	-4.35	-4.35	8.7	-4.35	-4.35	8.7	7.534421	-7.53442	-2.1E-15	7.534421	-7.53442
5	-11.5181	6.65	1.59E-14	-6.65	11.51814	-13.3	6.65	-11.5181	13.3	-11.5181	6.65
6	-13.8	13.8	-13.8	13.8	-13.8	13.8	1.69E-15	-3.4E-15	5.07E-15	-6.8E-15	3.3E-14
7	-13.943	8.05	-6.9E-15	-8.05	13.94301	-16.1	-8.05	13.94301	-16.1	13.94301	-8.05
8	-7.75	-7.75	15.5	-7.75	-7.75	15.5	-13.4234	13.42339	-7.6E-15	-13.4234	13.42339
9	-2.6E-15	-14.1	7.77E-15	14.1	1.21E-14	-14.1	-14.1	5.18E-15	14.1	-1E-14	-14.1
10	4.45	-4.45	-8.9	-4.45	4.45	8.9	-7.70763	-7.70763	2.13E-14	7.707626	7.707626
11	6.408588	3.7	8.16E-15	-3.7	-6.40859	-7.4	-3.7	-6.40859	-7.4	-6.40859	-3.7
12	3.6	3.6	3.6	3.6	3.6	3.6	-8.8E-16	-1.8E-15	-2.6E-15	-3.5E-15	-1.7E-14

a0	a1	a2	a3	a4	a5	a6	b1	b2	b3	b4	b5
9.466667	-5.28468	0.05	0.1	-0.51667	0.084679	-0.3	-3.81658	0.173205	0.5	-0.51962	-0.58342

ASSIGNMENT-1

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

$$a_0 = \frac{1}{12}\sum_{n=1}^{12} x(n), a_k = 2\times\frac{1}{12}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n, \quad b_k = 2\times\frac{1}{12}\sum_{n=1}^{12} x(n)\sin 2\pi f_k n$$

Month n	1	2	3	4	5	6	7	8	9	10	11	12
Average Temperature x(n)	3.4	4.5	4.3	8.7	13.3	13.8	16.1	15.5	14.1	8.9	7.4	3.6

n	$x(n)\cos 2\pi f_k n$						$x(n)\sin 2\pi f_k n$				
1	2.9445	1.7	2E-16	-1.7	-2.944	-3.4	1.7	2.9445	3.4	2.9445	1.7
2	2.25	-2.25	-4.5	-2.25	2.25	4.5	3.8971	3.8971	6E-16	-3.897	-3.897
3	3E-16	-4.3	-8E-16	4.3	5E-15	-4.3	4.3	5E-16	-4.3	-1E-15	4.3
4	-4.35	-4.35	8.7	-4.35	-4.35	8.7	7.5344	-7.534	-2E-15	7.5344	-7.534
5	-11.52	6.65	2E-14	-6.65	11.518	-13.3	6.65	-11.52	13.3	-11.52	6.65
6	-13.8	13.8	-13.8	13.8	-13.8	13.8	2E-15	-3E-15	5E-15	-7E-15	3E-14
7	-13.94	8.05	-7E-15	-8.05	13.943	-16.1	-8.05	13.943	-16.1	13.943	-8.05
8	-7.75	-7.75	15.5	-7.75	-7.75	15.5	-13.42	13.423	-8E-15	-13.42	13.423
9	-3E-15	-14.1	8E-15	14.1	1E-14	-14.1	-14.1	5E-15	14.1	-1E-14	-14.1
10	4.45	-4.45	-8.9	-4.45	4.45	8.9	-7.708	-7.708	2E-14	7.7076	7.7076
11	6.4086	3.7	8E-15	-3.7	-6.409	-7.4	-3.7	-6.409	-7.4	-6.409	-3.7
12	3.6	3.6	3.6	3.6	3.6	3.6	-9E-16	-2E-15	-3E-15	-4E-15	-2E-14

a0	a1	a2	a3	a4	a5	a6	b1	b2	b3	b4	b5
9.466666667	-5.285	0.05	0.1	-0.517	0.0847	-0.3	-3.817	0.1732	0.5	-0.52	-0.583

Yulubin
28j10103

Stu. ID: 28J10105

Month	Average Temperature
n	X(n)
1	3.4
2	4.5
3	4.3
4	8.7
5	13.3
6	13.8
7	16.1
8	15.5
9	14.1
10	8.9
11	7.4
12	3.6

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi f_k n + \sum_{k=1}^{5} b_k \sin 2\pi f_k n$$

$$a_0 = \frac{1}{12}\sum_{n=1}^{12} x(n),$$

$$a_k = 2 \times \frac{1}{12}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n$$

$$b_k = 2 \times \frac{1}{12}\sum_{n=1}^{12} x(n)\sin 2\pi f_k n$$

$$a_6 = \frac{1}{12}\sum_{n=1}^{12} x(n)\cos 2\pi f_k n$$

	cos(2πfkn)						sin(2πfkn)				
k	1	2	3	4	5	6	1	2	3	4	5
	0.866025404	0.5	6.13E-17	-0.5	-0.866025404	-1	0.5	0.866025	1	0.866025	0.5
	0.5	-0.5	-1	-0.5	0.5	1	0.866025	0.866025	1.23E-16	-0.86603	-0.86603
	6.12574E-17	-1	-1.8E-16	1	1.19447E-15	-1	1	1.23E-16	-1	-2.5E-16	1
	-0.5	-0.5	1	-0.5	-0.5	1	0.866025	-0.86603	-2.5E-16	0.866025	-0.86603
	-0.8660254	0.5	1.19E-15	-0.5	0.866025404	-1	0.5	-0.86603	1	-0.86603	0.5
	-1	1	-1	1	-1	1	1.23E-16	-2.5E-16	3.68E-16	-4.9E-16	2.39E-15
	-0.8660254	0.5	-4.3E-16	-0.5	0.866025404	-1	-0.5	0.866025	-1	0.866025	-0.5
	-0.5	-0.5	1	-0.5	-0.5	1	-0.86603	0.866025	-4.9E-16	-0.86603	0.866025
	-1.8377E-16	-1	5.51E-16	1	8.57495E-16	-1	-1	3.68E-16	1	-7.4E-16	-1
	0.5	-0.5	-1	-0.5	0.5	1	-0.86603	-0.86603	2.39E-15	0.866025	0.866025
	0.866025404	0.5	1.1E-15	-0.5	-0.866025404	-1	-0.5	-0.86603	-1	-0.86603	-0.5
	1	1	1	1	1	1	-2.5E-16	-4.9E-16	-7.4E-16	-9.8E-16	-4.8E-15

n	X(n)*cos(2πfkn)						X(n)*sin(2πfkn)				
1	2.944486373	1.7	2.08E-16	-1.7	-2.944486373	-3.4	1.7	2.944486	3.4	2.944486	1.7
2	2.25	-2.25	-4.5	-2.25	2.25	4.5	3.897114	3.897114	5.51E-16	-3.89711	-3.89711
3	2.63407E-16	-4.3	-7.9E-16	4.3	5.1362E-15	-4.3	4.3	5.27E-16	-4.3	-1.1E-15	4.3
4	-4.35	-4.35	8.7	-4.35	-4.35	8.7	7.534421	-7.53442	-2.1E-15	7.534421	-7.53442
5	-11.5181379	6.65	1.59E-14	-6.65	11.51813787	-13.3	6.65	-11.5181	13.3	-11.5181	6.65
6	-13.8	13.8	-13.8	13.8	-13.8	13.8	1.69E-15	-3.4E-15	5.07E-15	-6.8E-15	3.3E-14
7	-13.943009	8.05	-6.9E-15	-8.05	13.943009	-16.1	-8.05	13.94301	-16.1	13.94301	-8.05
8	-7.75	-7.75	15.5	-7.75	-7.75	15.5	-13.4234	13.42339	-7.6E-15	-13.4234	13.42339
9	-2.5912E-15	-14.1	7.77E-15	14.1	1.20907E-14	-14.1	-14.1	5.18E-15	14.1	-1E-14	-14.1
10	4.45	-4.45	-8.9	-4.45	4.45	8.9	-7.70763	-7.70763	2.13E-14	7.707626	7.707626
11	6.408587988	3.7	8.16E-15	-3.7	-6.408587988	-7.4	-3.7	-6.40859	-7.4	-6.40859	-3.7
12	3.6	3.6	3.6	3.6	3.6	3.6	-8.8E-16	-1.8E-15	-2.6E-15	-3.5E-15	-1.7E-14

a0	a1	a2	a3	a4	a5	a6	b1	b2	b3	b4	b5
9.466666667	-5.28467875	0.05	0.1	-0.51667	0.084678752	-0.3	-3.81658	0.173205	0.5	-0.51962	-0.58342

Name: Mao Bangcheng

Assignment of Theory of Dynamics and Control

By Erkang Fu 28J10107 Oct 30, 2010

Value of k and f_k						
k	1	2	3	4	5	6
f_k	0.0833	0.1667	0.2500	0.3333	0.4167	0.5000

$$x(n)=a_0+\sum_{k=1}^{6}a_k\cos 2\pi f_k n+\sum_{k=1}^{5}b_k\sin 2\pi f_k n$$

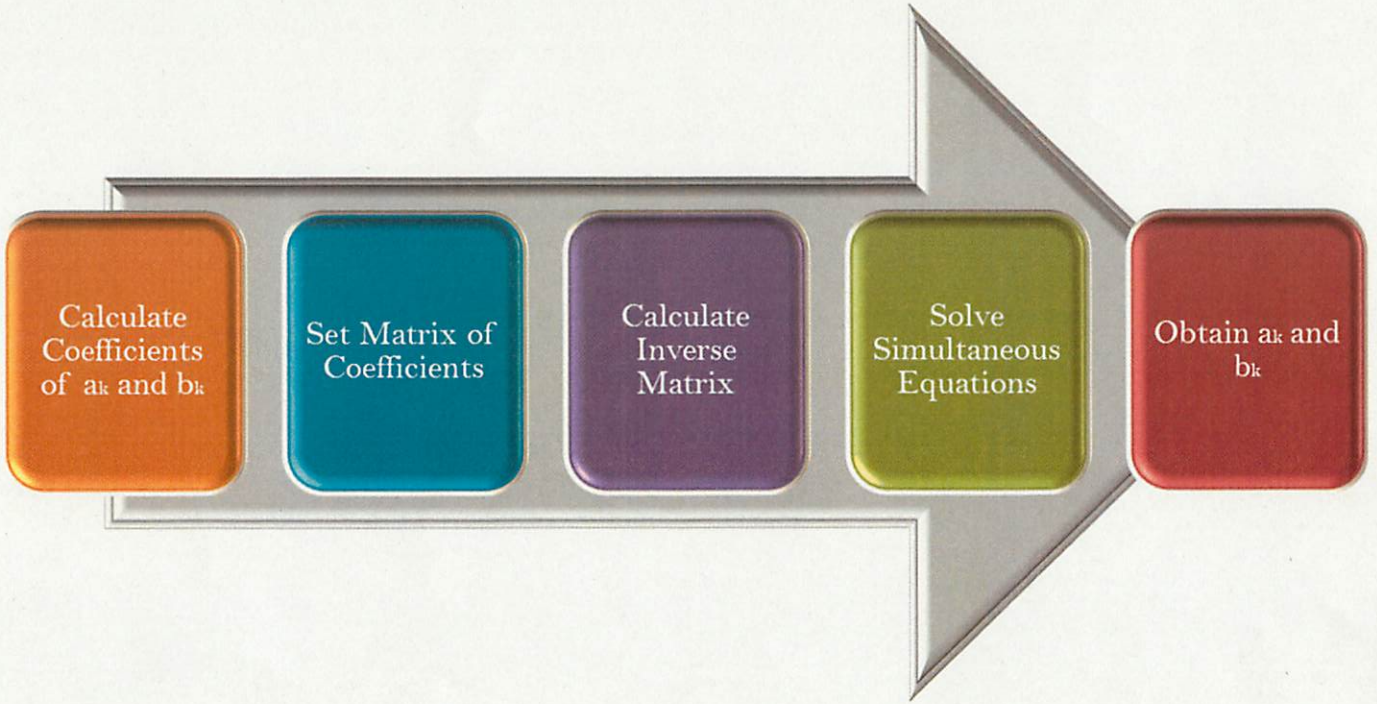
Matrix of a_k and b_k Coefficients

		cos(2 π f_kn)						sin(2 π f_kn)					Average Temperature	Checking Calculation
n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	x(n)	
1	1.0000	0.8660	0.5000	0.0000	-0.5000	-0.8660	-1.0000	0.5000	0.8660	1.0000	0.8660	0.5000	3.40	TRUE
2	1.0000	0.5000	-0.5000	-1.0000	-0.5000	0.5000	1.0000	0.8660	0.8660	0.0000	-0.8660	-0.8660	4.50	TRUE
3	1.0000	0.0000	-1.0000	0.0000	1.0000	0.0000	-1.0000	1.0000	0.0000	-1.0000	0.0000	1.0000	4.30	TRUE
4	1.0000	-0.5000	-0.5000	1.0000	-0.5000	-0.5000	1.0000	0.8660	-0.8660	0.0000	0.8660	-0.8660	8.70	FALSE
5	1.0000	-0.8660	0.5000	0.0000	-0.5000	0.8660	-1.0000	0.5000	-0.8660	1.0000	-0.8660	0.5000	13.30	TRUE
6	1.0000	-1.0000	1.0000	-1.0000	1.0000	-1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	13.80	TRUE
7	1.0000	-0.8660	0.5000	0.0000	-0.5000	0.8660	-1.0000	-0.5000	0.8660	-1.0000	0.8660	-0.5000	16.10	TRUE
8	1.0000	-0.5000	-0.5000	1.0000	-0.5000	-0.5000	1.0000	-0.8660	0.8660	0.0000	-0.8660	0.8660	15.50	TRUE
9	1.0000	0.0000	-1.0000	0.0000	1.0000	0.0000	-1.0000	-1.0000	0.0000	1.0000	0.0000	-1.0000	14.10	TRUE
10	1.0000	0.5000	-0.5000	-1.0000	-0.5000	0.5000	1.0000	-0.8660	-0.8660	0.0000	0.8660	0.8660	8.90	TRUE
11	1.0000	0.8660	0.5000	0.0000	-0.5000	-0.8660	-1.0000	-0.5000	-0.8660	-1.0000	-0.8660	-0.5000	7.40	TRUE
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.60	TRUE

Inverse Matrix of a_k and b_k Coefficients

0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	3.40
0.1443	0.0833	0.0000	-0.0833	-0.1443	-0.1667	-0.1443	-0.0833	0.0000	0.0833	0.1443	0.1667	4.50
0.0833	-0.0833	-0.1667	-0.0833	0.0833	0.1667	0.0833	-0.0833	-0.1667	-0.0833	0.0833	0.1667	4.30
0.0000	-0.1667	0.0000	0.1667	0.0000	-0.1667	0.0000	0.1667	0.0000	-0.1667	0.0000	0.1667	8.70
-0.0833	-0.0833	0.1667	-0.0833	-0.0833	0.1667	-0.0833	-0.0833	0.1667	-0.0833	-0.0833	0.1667	13.30
-0.1443	0.0833	0.0000	-0.0833	0.1443	-0.1667	0.1443	-0.0833	0.0000	0.0833	-0.1443	0.1667	13.80
-0.0833	0.0833	-0.0833	0.0833	-0.0833	0.0833	-0.0833	0.0833	-0.0833	0.0833	-0.0833	0.0833	16.10
0.0833	0.1443	0.1667	0.1443	0.0833	0.0000	-0.0833	-0.1443	-0.1667	-0.1443	-0.0833	0.0000	15.50
0.1443	0.1443	0.0000	-0.1443	-0.1443	0.0000	0.1443	0.1443	0.0000	-0.1443	-0.1443	0.0000	14.10
0.1667	0.0000	-0.1667	0.0000	0.1667	0.0000	-0.1667	0.0000	0.1667	0.0000	-0.1667	0.0000	8.90
0.1443	-0.1443	0.0000	0.1443	-0.1443	0.0000	0.1443	-0.1443	0.0000	0.1443	-0.1443	0.0000	7.40
0.0833	-0.1443	0.1667	-0.1443	0.0833	0.0000	-0.0833	0.1443	-0.1667	0.1443	-0.0833	0.0000	3.60

a_0	**9.47**
a_1	**-5.28**
a_2	**0.05**
a_3	**0.10**
a_4	**-0.52**
a_5	**0.08**
a_6	**-0.30**
b_1	**-3.82**
b_2	**0.17**
b_3	**0.50**
b_4	**-0.52**
b_5	**-0.58**



Yaseen Adnan Ahmed

Studuent ID: 28J10108

Assignment No1: Calculate the Cosine & Sine coefficient of Fourier series from the given sample data.

Given Data:

n	x(n)
1	3.4
2	4.5
3	4.3
4	8.7
5	13.3
6	13.8
7	16.1
8	15.5
9	14.1
10	8.9
11	7.4
12	3.6

Solution: Formulas used for the calculation are given below.....

$$a_k = 2/N^* \Sigma(x(n)\ Cos2\Pi^* f_k^* n) \quad \text{for } k=1,2,3,4,5$$

$$a_k = 1/N^* \Sigma(x(n)\ Cos2\Pi^* f_k^* n) \quad \text{for } k=6$$

$$b_k = 2/N^* \Sigma(x(n)\ Sin2\Pi^* f_k^* n) \quad \text{for } k=1,2,3,4,5$$

where, $f_k = K/12$

N = 12 , Number of sample data.

Calculation for a_k

K / n	1	2	3	4	5	6
1	0.490747729	0.283333	3.47125E-17	-0.28333333	-0.49075	-0.28333333
2	0.375	-0.375	-0.75	-0.375	0.375	0.375
3	4.39012E-17	-0.71667	-1.317E-16	0.716666667	2.2E-16	-0.35833333
4	-0.725	-0.725	1.45	-0.725	-0.725	0.725
5	-1.919689645	1.108333	6.78936E-16	-1.10833333	1.91969	-1.10833333
6	-2.3	2.3	-2.3	2.3	-2.3	1.15
7	-2.323834833	1.341667	-1.1506E-15	-1.34166667	2.323835	-1.34166667
8	-1.291666667	-1.29167	2.583333333	-1.29166667	-1.29167	1.291666667
9	-4.31865E-16	-2.35	1.29559E-15	2.35	2.02E-15	-1.175
10	0.741666667	-0.74167	-1.48333333	-0.74166667	0.741667	0.741666667
11	1.068097998	0.616667	-3.0219E-15	-0.61666667	-1.0681	-0.61666667
12	0.6	0.6	0.6	0.6	0.6	0.3
Sum=	-5.284678752	0.05	0.1	-0.51666667	0.084679	-0.3

Here the last low gives the summation of each column and our expected results for Cosine coefficient.

Calculation for b_k

K / N	1	2	3	4	5
1	0.283333	0.490748	0.566667	0.490748	0.283333
2	0.649519	0.649519	9.19E-17	-0.64952	-0.64952
3	0.716667	8.78E-17	-0.71667	-1.8E-16	0.716667
4	1.255737	-1.25574	-3.6E-16	1.255737	-1.25574
5	1.108333	-1.91969	2.216667	-1.91969	1.108333
6	2.82E-16	-5.6E-16	8.45E-16	-1.1E-15	1.41E-15
7	-1.34167	2.323835	-2.68333	2.323835	-1.34167
8	-2.23723	2.237232	-1.3E-15	-2.23723	2.237232
9	-2.35	8.64E-16	2.35	-1.7E-15	-2.35
10	-1.2846	-1.2846	9.09E-16	1.284604	1.284604
11	-0.61667	-1.0681	-1.23333	-1.0681	-0.61667
12	-1.5E-16	-2.9E-16	-4.4E-16	-5.9E-16	-7.4E-16
Sum=	-3.81658	0.173205	0.5	-0.51962	-0.58342

Here the last low gives the summation of each column and our expected results for Sine co-efficient.

Summary of the results:

K	1	2	3	4	5	6
a_k	-5.284678752	0.05	0.1	-0.51666667	0.084679	-0.3
b_k	-3.816580754	0.173205	0.5	-0.51961524	-0.58342	-

Theory of Dynamics and Control **01.11.2010**
Prof. Hasegawa
Assignment I

Tokgoz Emel
28J10109

The Discrete Fourier Expansion

For the equally discrete series x(n), (n=1,2,…,12), the following expression is;

$$x(n) = a_0 + \sum_{k=1}^{6} a_k \cos 2\pi fkn + \sum_{k=1}^{5} b_k \sin 2\pi fkn$$

where T=12,Δt=1 and fk is frequency and equal to k/12.
The coefficients for this expression are;

$$a_0 = \frac{1}{T}\sum_{n=1}^{12} x\,(n.\Delta t).\Delta t$$

$$a_k = \frac{2}{T}\int_{n=1}^{T} x(n).\cos\,(2\,\pi nk/12).dn$$

$$b_k = \frac{2}{T}\int_{n=1}^{T} x(n).\sin\,(2\,\pi nk/12).dn$$

The coefficients that were found discretely are shown below;

$a_0 = 9.466666667$
$a_1 = -5.284678752$
a_2=0.05
$a_3 = 0.1$
$a_4 = -0.516666667$
a_5=0.084678752
a_6=-0.6

b_1=-3.816580754
b_2=0.173205081
b_3=0.5
b_4=-0.519615242
b_5=-0.583419246

Δt=T/2n	1	**Coefficients**		
n=	6	**a0**	average value	**9.466666667**
T=	12	ak	cosine coefficients	
fk=k/T	k/12	bk	sine coefficients	
2/T=	0.167			

		a1	a2	a3	a4	a5	a6
n	x(n)	x(n).cos(30n)	x(n).cos(30.2n)	x(n).cos(30.3n)	x(n).cos(30.4n)	x(n).cos(30.5n)	x(n).cos(30.6n)
1	3.4	2.944486373	1.7	2.08275E-16	-1.7	-2.944486373	-3.4
2	4.5	2.25	-2.25	-4.5	-2.25	2.25	4.5
3	4.3	2.63407E-16	-4.3	-7.90221E-16	4.3	1.31703E-15	-4.3
4	8.7	-4.35	-4.35	8.7	-4.35	-4.35	8.7
5	13.3	-11.51813787	6.65	4.07362E-15	-6.65	11.51813787	-13.3
6	13.8	-13.8	13.8	-13.8	13.8	-13.8	13.8
7	16.1	-13.943009	8.05	-6.90371E-15	-8.05	13.943009	-16.1
8	15.5	-7.75	-7.75	15.5	-7.75	-7.75	15.5
9	14.1	-2.59119E-15	-14.1	7.77357E-15	14.1	1.20907E-14	-14.1
10	8.9	4.45	-4.45	-8.9	-4.45	4.45	8.9
11	7.4	6.408587988	3.7	-1.81314E-14	-3.7	-6.408587988	-7.4
12	3.6	3.6	3.6	3.6	3.6	3.6	3.6
Total	area	-31.70807251	0.3	0.6	-3.1	0.50807251	-3.6
	ak	**-5.284678752**	**0.05**	**0.1**	**-0.516666667**	**0.084678752**	**-0.6**

		b1	b2	b3	b4	b5
n	x(n)	x(n).sin(30n)	x(n).sin(30.2n)	x(n).sin(30.3n)	x(n).sin(30.4n)	x(n).sin(30.5n)
1	3.4	1.7	2.944486373	3.4	2.944486373	1.7
2	4.5	3.897114317	3.897114317	5.51317E-16	-3.897114317	-3.897114317
3	4.3	4.3	5.26814E-16	-4.3	-1.05363E-15	4.3
4	8.7	7.534421013	-7.534421013	-2.13176E-15	7.534421013	-7.534421013
5	13.3	6.65	-11.51813787	13.3	-11.51813787	6.65
6	13.8	1.6907E-15	-3.38141E-15	5.07211E-15	-6.76282E-15	8.45352E-15
7	16.1	-8.05	13.943009	-16.1	13.943009	-8.05
8	15.5	-13.42339376	13.42339376	-7.59592E-15	-13.42339376	13.42339376
9	14.1	-14.1	5.18238E-15	14.1	-1.03648E-14	-14.1
10	8.9	-7.707626094	-7.707626094	5.45191E-15	7.707626094	7.707626094
11	7.4	-3.7	-6.408587988	-7.4	-6.408587988	-3.7
12	3.6	-8.82107E-16	-1.76421E-15	-2.64632E-15	-3.52843E-15	-4.41053E-15
Total	area	-22.89948452	1.039230485	3	-3.117691454	-3.500515478
	bk	**-3.816580754**	**0.173205081**	**0.5**	**-0.519615242**	**-0.583419246**

MA CHONG

28J10111

		K	1	2	3	4	5	6		1	2	3	4	5	6
1	3.4		0.866025	0.5	6.12574E-17	-0.5	-0.86603	-1		0.5	0.866025	1	0.866025	0.5	1.22515E-16
2	4.5		0.5	-0.5	-1	-0.5	0.5	1		0.866025	0.866025	1.23E-16	-0.86603	-0.86603	-2.4503E-16
3	4.3		6.13E-17	-1	-1.83772E-16	1	3.06E-16	-1		1	1.23E-16	-1	-2.5E-16	1	3.67545E-16
4	8.7		-0.5	-0.5	1	-0.5	-0.5	1		0.866025	-0.86603	-2.5E-16	0.866025	-0.86603	-4.90059E-16
5	13.3		-0.86603	0.5	3.06287E-16	-0.5	0.866025	-1		0.5	-0.86603	1	-0.86603	0.5	6.12574E-16
6	13.8		-1	1	-1	1	-1	1		1.23E-16	-2.5E-16	3.68E-16	-4.9E-16	6.13E-16	-7.35089E-16
7	16.1		-0.86603	0.5	-4.28802E-16	-0.5	0.866025	-1		-0.5	0.866025	-1	0.866025	-0.5	8.57604E-16
8	15.5		-0.5	-0.5	1	-0.5	-0.5	1		-0.86603	0.866025	-4.9E-16	-0.86603	0.866025	-9.80119E-16
9	14.1		-1.8E-16	-1	5.51317E-16	1	8.57E-16	-1		-1	3.68E-16	1	-7.4E-16	-1	1.10263E-15
10	8.9		0.5	-0.5	-1	-0.5	0.5	1		-0.86603	-0.86603	6.13E-16	0.866025	0.866025	-1.22515E-15
11	7.4		0.866025	0.5	-2.45019E-15	-0.5	-0.86603	-1		-0.5	-0.86603	-1	-0.86603	-0.5	4.90038E-15
12	3.6		1	1	1	1	1	1		-2.5E-16	-4.9E-16	-7.4E-16	-9.8E-16	-1.2E-15	-1.47018E-15
			2.944486	1.7	2.08275E-16	-1.7	-2.94449	-3.4		1.7	2.944486	3.4	2.944486	1.7	4.1655E-16
			2.25	-2.25	-4.5	-2.25	2.25	4.5		3.897114	3.897114	5.51E-16	-3.89711	-3.89711	-1.10263E-15
			2.63E-16	-4.3	-7.90221E-16	4.3	1.32E-15	-4.3		4.3	5.27E-16	-4.3	-1.1E-15	4.3	1.58044E-15
			-4.35	-4.35	8.7	-4.35	-4.35	8.7		7.534421	-7.53442	-2.1E-15	7.534421	-7.53442	-4.26352E-15
			-11.5181	6.65	4.07362E-15	-6.65	11.51814	-13.3		6.65	-11.5181	13.3	-11.5181	6.65	8.14724E-15
			-13.8	13.8	-13.8	13.8	-13.8	13.8		1.69E-15	-3.4E-15	5.07E-15	-6.8E-15	8.45E-15	-1.01442E-14
			-13.943	8.05	-6.90371E-15	-8.05	13.94301	-16.1		-8.05	13.94301	-16.1	13.94301	-8.05	1.38074E-14
			-7.75	-7.75	15.5	-7.75	-7.75	15.5		-13.4234	13.42339	-7.6E-15	-13.4234	13.42339	-1.51918E-14
			-2.6E-15	-14.1	7.77357E-15	14.1	1.21E-14	-14.1		-14.1	5.18E-15	14.1	-1E-14	-14.1	1.55471E-14
			4.45	-4.45	-8.9	-4.45	4.45	8.9		-7.70763	-7.70763	5.45E-15	7.707626	7.707626	-1.09038E-14
			6.408588	3.7	-1.81314E-14	-3.7	-6.40859	-7.4		-3.7	-6.40859	-7.4	-6.40859	-3.7	3.62628E-14
			3.6	3.6	3.6	3.6	3.6	3.6		-8.8E-16	-1.8E-15	-2.6E-15	-3.5E-15	-4.4E-15	-5.29264E-15
	113.6		-31.7081	0.3	0.6	-3.1	0.508073	-3.6		-22.8995	1.03923	3	-3.11769	-3.50052	2.88629E-14

a0		a1	a2	a3	a4	a5	a6		b1	b2	b3	b4	b5	b6
9.466667		-5.28468	0.05	0.1	-0.51667	0.084679	-0.6		-3.81658	0.173205	0.5	-0.51962	-0.58342	4.81048E-15

[Theory of Motion and Control] Homework of 18/10

28j10004 Yuto Ito

k	fk	x(n)	n
1	0.083333	3.4	1
2	0.166667	4.5	2
3	0.25	4.3	3
4	0.333333	8.7	4
5	0.416667	13.3	5
6	0.5	13.8	6
7	0.583333	16.1	7
8	0.666667	15.5	8
9	0.75	14.1	9
10	0.833333	8.9	10
11	0.916667	7.4	11
12	1	3.6	12

	ak	bk
0	9.466667	
1	-5.28468	-3.816580754
2	0.05	0.173205081
3	0.1	0.5
4	-0.51667	-0.519615242
5	0.084679	-0.583419246
6	-0.3	

x(n)cos2 π fkn

k=1	2	3	4	5	6
2.944486	1.7	2.08275E-16	-1.7	-2.944486373	-3.4
2.25	-2.25	-4.5	-2.25	2.25	4.5
2.63E-16	-4.3	-7.90221E-16	4.3	1.31703E-15	-4.3
-4.35	-4.35	8.7	-4.35	-4.35	8.7
-11.5181	6.65	4.07362E-15	-6.65	11.51813787	-13.3
-13.8	13.8	-13.8	13.8	-13.8	13.8
-13.943	8.05	-6.90371E-15	-8.05	13.943009	-16.1
-7.75	-7.75	15.5	-7.75	-7.75	15.5
-2.6E-15	-14.1	7.77357E-15	14.1	1.20907E-14	-14.1
4.45	-4.45	-8.9	-4.45	4.45	8.9
6.408588	3.7	-1.81314E-14	-3.7	-6.408587988	-7.4
3.6	3.6	3.6	3.6	3.6	3.6

x(n)sin2 π fkn

k=1	2	3	4	5
1.7	2.944486373	3.4	2.944486	1.7
3.897114	3.897114317	5.51E-16	-3.89711	-3.89711
4.3	5.26814E-16	-4.3	-1.1E-15	4.3
7.534421	-7.534421013	-2.1E-15	7.534421	-7.53442
6.65	-11.51813787	13.3	-11.5181	6.65
1.69E-15	-3.38141E-15	5.07E-15	-6.8E-15	8.45E-15
-8.05	13.943009	-16.1	13.94301	-8.05
-13.4234	13.42339376	-7.6E-15	-13.4234	13.42339
-14.1	5.18238E-15	14.1	-1E-14	-14.1
-7.70763	-7.707626094	5.45E-15	7.707626	7.707626
-3.7	-6.408587988	-7.4	-6.40859	-3.7
-8.8E-16	-1.76421E-15	-2.6E-15	-3.5E-15	-4.4E-15