

**ESTIMATION OF MATHEMATICAL MODEL AND ITS COEFFICIENTS OF SHIP MANOEUVRABILITY FOR A TWIN-PROPELLER TWIN-RUDDER SHIP**

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**Abstract:** Mathematical model for ship manoeuvrability is the most important issue for a ship manoeuvring simulation. There is well-established MMG model developed in Japan since 1970s for single-propeller single-rudder ships and some database or estimation charts are available for given ships requested for manoeuvring studies. The model is also expanded for shallow water and some data are available for a few ships, but they are not enough to estimate for any given unknown ship. Furthermore, there are not yet well-established model for twin-propeller twin-rudder ships. In this paper, some discussion on a mathematical model for twin-propeller twin-rudder will be done and an attempt to estimate its coefficients will be introduced.

**1. INTRODUCTION**

MMG model is well known as a mathematical model for ship manoeuvrability. Although it is originally designated for single-propeller single-rudder ships, some researchers have attempted and succeeded to expand it for other types of ships. However, it is still very hard to estimate the hydrodynamic coefficients, especially interaction coefficients, without conducting model experiments. Furthermore, conventional empirical formulae to estimate them such as Kijima's regression model are not properly suitable for non-conventional ships, because of lack of sufficient mother data.

In this research the manoeuvring characteristics of a twin-propeller twin-rudder ship were firstly reviewed in details. Moreover, as no experimental data concerning the subject ship were available, a method for estimating the coefficients is proposed in order to identify the mathematical model of the subject ship referring to the manoeuvring study of other multi-propeller multi-rudder model ships developed earlier by other researchers.

Finally, the predicted manoeuvring performance of the subject ship is discussed and compared with the actual sea trials results. Moreover a regression model numerical simulations under steady wind conditions were also carried out.

**2. TWIN-PROPELLER TWIN-RUDDER SHIP MATHEMATICAL MODEL**

Three degrees of freedom model consisting of surge sway and yaw motion is considered. The coordinate system for the twin propeller twin rudder ship is shown in Fig.1.

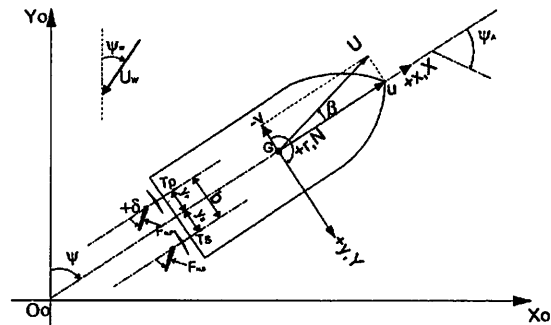


Fig. 1 Coordinate system with body fixed axis at ship's center of gravity.

The equations of surge, sway and yaw motion considering the origin of coordinate system at the ship center of gravity can be basically written as follows in equation (1):

$$\begin{aligned} (m + m_x)\dot{u} - (m + m_y)vr &= X_H + X_P + X_R \\ (m + m_y)\dot{v} + (m + m_x)ur &= Y_H + Y_P + Y_R \\ (I_z + J_{zz})\dot{r} &= N_H + N_P + N_R \end{aligned} \quad (1)$$

On the right hand side, the subscripts *H*, *P* and *R* stand for force and moment due to hull, propeller and rudder respectively.

Different researches were done and different models were proposed for twin-propeller twin-rudder ships among which the following ones were analyzed:

**2.1 Kobayashi's model [1]:**

$$v_p' = -\sin \beta + x_p' r'$$

$$x_p' = \frac{x_p}{L} \quad (13)$$

$$r' = \frac{r}{\left(\frac{U}{L}\right)}$$

$U$ ,  $L$ ,  $x_p$ , and  $w_{p0}$  correspond respectively to ship speed, ship length, location of propeller in  $x$  axis direction and propeller wake in ahead motion.

### 3. PREDICTION METHOD OF SUBJECT SHIP HYDRODYNAMIC COEFFICIENTS

The subject ship is a twin-propeller twin-rudder car ferry with a hard chin and a transom stern whose principle particulars are shown in Table.1. The ship body plan is also shown in Fig.2. It has to be noted that there is neither experiment nor research for the subject ship or similar ships done or published before.

Table 1 Subject ship principal particulars

$L$	49.00	$D_p$	1.45
$B$	14.00	$b_p$	8.80
$d$	2.65	$P/D_p$	0.80
$C_b$	0.63	$D_p/H_R$	0.87

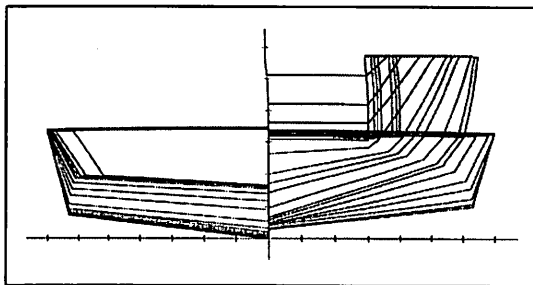


Fig. 3 Subject ship body plan

#### 3.1 Hull forces :

The subject ship mathematical model was based originally on Lee's model. The hull forces were estimated using Kang *et al.* [5] regression method. The equations expressing these hull forces for surge, sway and yaw motion are written in the following equations:

$$X'' = (a_2 \cdot \sin^2(\beta) + a_4 \cdot \sin^2(2\beta)) \cdot \cos(\beta)$$

$$+ b_1 \cdot \sin(\beta) \cdot r' + b_2 \cdot \sin(2\beta) \cdot r' \cdot \text{sign}(\cos(\beta))$$

$$Y'' = (a_1 + c_1 \cdot r'^2) \cdot \sin(\beta) + a_3 \cdot \sin(3\beta)$$

$$+ a_5 \cdot \sin(5\beta) + (d_1 \cdot r' + e_1 \cdot r'') \cdot \cos(\beta) \quad (14)$$

$$Y'' = (a_1 + c_1 \cdot r'^2) \cdot \sin(\beta) + a_3 \cdot \sin(3\beta)$$

$$+ a_5 \cdot \sin(5\beta) + (d_1 \cdot r' + e_1 \cdot r'') \cdot \cos(\beta)$$

#### 3.2 Added mass :

The added mass and moment of inertia were estimated by Motora *et al.* [6] diagrams. The moment of inertia  $I_{ZZ}$  was estimated as is written in Eqs. (15) and (16).

$$K_{ZZ} = -0.2536 \cdot L \quad (15)$$

$$I_{ZZ} = m \cdot K_{ZZ}^2 \quad (16)$$

Where  $K_{ZZ}$  corresponds to the radius of gyration.

#### 3.3 Ship resistance performance :

The subject ship resistance performance was estimated from similar hull type ship resistance test data as follows in Eq. (17).

$$R_r = 0.5 \rho U^3 S \{ (1+k) C_{F0} + C_w + C_{BR} + \Delta C_F \} \quad (17)$$

Where  $U$ : ship velocity,  $S$ : wetted surface area,  $k$ : form factor,  $C_{F0}$ : frictional coefficient,  $C_w$ : wave making resistance,  $C_{BR}$ : shaft brackets effect coefficient,  $\Delta C_F$ : skin roughness coefficient.

Both coefficients  $k$  and  $\Delta C_F$  were estimated respectively as 0.15 and 0.005 (Toda, 2008) [7]. The frictional coefficient  $C_{F0}$  was calculated according to the ITTC chart (Lewis 1988) [8] as shown in Eq. (18).

$$C_{F0} = \frac{0.075}{(\log_{10} R_n - 2)^2} \quad (18)$$

Where  $R_n$ : Reynolds number.

The wave making resistance  $C_w$  and the shaft brackets effect coefficient  $C_{BR}$  were determined from the experimental data of similar hull type ship and then they were tuned to fit the subject ship using its principal particulars dimensions.

#### 3.4 Propellers and rudders coefficients :

The mathematical model for the propellers thrust is expressed as in equations (19-21)

$$\begin{aligned} X_H &= X_{uu} u^2 + X_{vv} v^2 + X_{www} v^4 + X_{rr} r^2 + X_{vr} vr \\ Y_H &= Y_v v + Y_{vv} v^3 + Y_r r + Y_{rr} r^3 + Y_{vr} v^2 r + Y_{vr} vr^2 \quad (2) \\ N_H &= N_v v + N_{vv} v^3 + N_r r + N_{rr} r^3 + N_{vr} v^2 r \\ &\quad + N_{vr} vr^2 \end{aligned}$$

$$\begin{aligned} X_P &= (1-t)(T_{(s)} + T_{(p)}) \\ Y_P &= \Delta Y_P = \Delta Y_{P(s)}(J_s) - \Delta Y_{P(p)}(J_s) \\ N_P &= (1-t) \frac{b_P}{2} (T_{(s)} - T_{(p)}) \\ &\quad + (\Delta N_{P(p)}(J_s) + \Delta N_{P(p)}(J_s)) \quad (3) \end{aligned}$$

$$\begin{aligned} X_R &= -\{(1-t_r)(F_{N(s)} \sin \delta_{(s)} + F_{N(p)} \sin \delta_{(p)})\} \\ Y_R &= -\{(1+a_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ N_R &= -\{(x_R + a_H x_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ &\quad - \frac{b_P}{2} (1-t_r)(F_{N(p)} \sin \delta_{(p)} - F_{N(s)} \sin \delta_{(s)}) \quad (4) \end{aligned}$$

The subscripts *H*, *P*, *R*, *N*, *s* and *p* stand for (force or moment due to) hull, propeller, rudder, starboard and port respectively throughout in this paper. The terms  $\Delta Y_P$ , and  $\Delta N_P$  correspond respectively to the resultant sway force and the added yaw moment due to the interaction forces between both propellers.

2.2 Yoshimura's model [2]:

$$\begin{aligned} X_H &= X_0 + X_{\beta\beta} \beta^2 + X_{\beta r} \beta r + X_{rr} r^2 + X_{\beta\beta\beta} \beta^3 r + X_{\beta\beta\beta} \beta^4 \\ Y_H &= Y_\beta \beta + Y_r r + Y_{\beta\beta} \beta^3 + Y_{\beta\beta} \beta^2 r + Y_{\beta r} \beta r^2 \\ N_H &= N_\beta \beta + N_r r + N_{\beta\beta} \beta^3 + N_{\beta\beta} \beta^2 r + N_{\beta r} \beta r^2 + N_{rr} r^3 \\ X_P &= (1-t)(T_{(s)} + T_{(p)}) \\ N_P &= (1-t) \frac{b_P}{2} (T_{(p)} - T_{(s)}) \quad (6) \end{aligned}$$

$$\begin{aligned} X_R &= -\{1-t_r - (1-t)C_{\delta\delta}\} (F_{N(s)} \sin \delta_{(s)} + F_{N(p)} \sin \delta_{(p)}) \\ Y_R &= -\{(1+a_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ N_R &= -\{(x_R + a_H x_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ &\quad - \frac{b_P}{2} \{1-t_r - (1-t)C_{\delta\delta}\} (F_{N(p)} \sin \delta_{(p)} - F_{N(s)} \sin \delta_{(s)}) \quad (7) \end{aligned}$$

The term  $C_{\delta\delta}$  corresponds to the moment caused by the difference of the longitudinal component of rudder force.

2.3 Lee's model [3,4]:

In this model, the origin of coordinate system is shown in Fig.2. The ship equation of motion is therefore written as follows

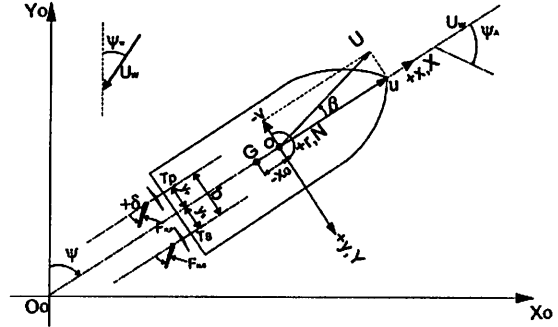


Fig. 2 Coordinate system with body fixed axis at ship's midship.

$$\begin{aligned} (m + m_x) \dot{u} - (m + m_y) vr - m x_G r^2 &= X_H + X_P + X_R \\ (m + m_y) \dot{v} + (m x_G - Y \dot{r}) \dot{r} + (m + m_x) ur &= Y_H + Y_P + Y_R \quad (8) \\ (I_z + J_z) \dot{r} + (m x_G - N \dot{v}) \dot{v} + m x_G ur &= N_H + N_P + N_R \end{aligned}$$

The hydrodynamic forces and moment acting on hull, propeller and rudder are written as in Eqs. (9)–(11) respectively

$$\begin{aligned} X_H &= X_{vv} v^2 + X_{www} v^4 + X_{vr} vr + X_{rr} r^2 \\ Y_H &= Y_v v + Y_r r + Y_{vv} v^3 + Y_{vr} v^2 r + Y_{vr} vr^2 + Y_{rr} r^3 \quad (9) \\ N_H &= N_v v + N_r r + N_{vv} v^3 + N_{vr} v^2 r + N_{vr} vr^2 \\ &\quad + N_{rr} r^3 \end{aligned}$$

$$\begin{aligned} X_P &= (1-t)(T_{(s)} + T_{(p)}) \\ N_P &= (1-t) \frac{b_P}{2} (T_{(p)} - T_{(s)}) \quad (10) \end{aligned}$$

$$\begin{aligned} X_R &= -\{(1-t_r)(F_{N(s)} \sin \delta_{(s)} + F_{N(p)} \sin \delta_{(p)})\} \\ Y_R &= -\{(1+a_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ N_R &= -\{(x_R + a_H x_H)(F_{N(s)} \cos \delta_{(s)} + F_{N(p)} \cos \delta_{(p)})\} \\ &\quad - \frac{b_P}{2} (1-t_r)(F_{N(p)} \sin \delta_{(p)} - F_{N(s)} \sin \delta_{(s)}) \quad (11) \end{aligned}$$

In this study, Lee's model was chosen as the basic model for the subject ship. The reason is that the principal particulars of Lee's ship model is similar to the subject ship, indeed all coefficients concerning the model were available as well as a proposed regression equation for calculating propeller wake during manoeuvring.

$$\begin{aligned} (1 - w_{p[s]}) &= (1 - w_{p0}) \\ &\quad + \tau_{[s]} \left\{ (v_p^4 + C_{p[s]} v_p^3 |v_p|^2) + c_{pv[s]} v^3 + c_{pr[s]} r^3 \right\} \quad (12) \end{aligned}$$

Where

$$X_{\{s\}P} = (1 - t_{\{s\}P}) \rho n^2 D_p^4 K_{\{s\}T}(J_{\{s\}P}) \quad (19)$$

$$N_{\{s\}P} = (1 - t_{\{s\}P}) \frac{b_p}{2} (\rho n^2 D_p^4 K_{\{s\}T}(J_{\{s\}P}) - \rho n^2 D_p^4 K_{\{s\}T}(J_{\{s\}P})) \quad (20)$$

$$\left. \begin{aligned} K_{\{s\}T}(J_{\{s\}P}) &= C_1 + C_2 J_{\{s\}P} + C_3 J_{\{s\}P}^2 \\ J_{\{s\}P} &= u(1 - w_{\{s\}P}) / (n D_p) \end{aligned} \right\} \quad (21)$$

The coefficients of thrust deduction fraction  $t_p$  and wake fraction  $w_{p0}$  were not available and then needed to be estimated as written in Table.2 (Toda, 2008) [7]. The coefficients of propeller thrust in open water were also not available and were estimated by selecting a Wageningen - B series propeller (Kuiper 1992) [9] whose principal characteristics (number of blades, pitch ratio and expanded area ratio) were similar to the propeller fitted on the subject ship, for the operating range of propeller at cruising and manoeuvring speed.

Table 2 Propeller coefficients

$t_p$	0.20
$w_{p0}$	0.10
$C_1$	0.3721
$C_2$	-0.3142
$C_3$	0.1594

The rudder drag coefficient was estimated as  $t_R = 0.3$  and the coefficients of interaction force between rudder and ship hull were calculated based on a method proposed by Lee *et al.* [3,4] as follows

$$\begin{aligned} a_H &= 0.35 \\ x_H &= -0.4 \cdot L \end{aligned} \quad (22)$$

The rudder normal force is expressed as shown in Eq. (23).

$$F_N = \frac{\rho}{2} A_R U_R^2 f_\alpha \cdot \sin \alpha_R \quad (23)$$

Where  $A_R$ : rudder area,  $U_R$ : inflow velocity to the rudder, and  $\alpha_R$ : effective inflow angle to the rudder.

#### 4. SIMULATION (FIRST TRIAL)

After determining all the hydrodynamic coefficients, simulation of full scale turning tests was carried out and some results are shown in Figs.4 and 5.

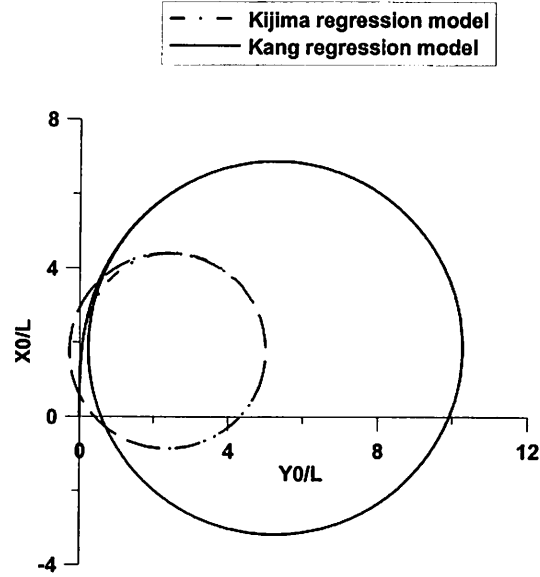


Fig. 4 Simulation of starboard turning ( $\delta\{s,p\}=5^\circ$ )

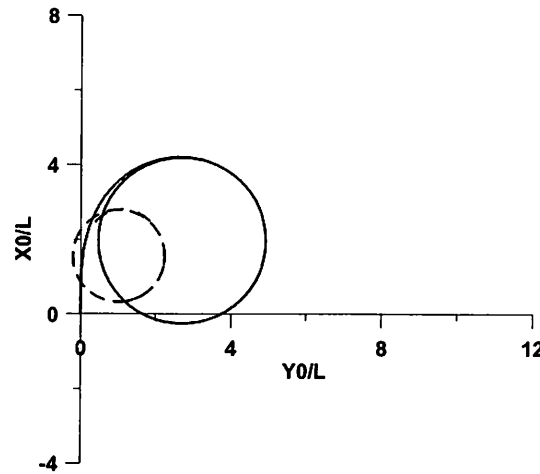


Fig. 5 Simulation of starboard turning ( $\delta\{s,p\}=15^\circ$ )

A first set of simulation was done based on Kang's regression model (Kang *et al.* [5]) for estimating the hydrodynamic forces acting on the hull. The estimated hydrodynamic hull forces showed quite reasonable and acceptable results for large turning angle as it is shown in Fig. 4. However, the more the rudder angle is increased, the more the results seem to be not appropriate and completely irrelevant for 15 degrees turning as shown in Fig. 5.

A second set of simulation was done based on Kijima's regression model (Kijima *et al.* [10]) for hull hydrodynamic forces. It can be said that it is inappropriate for the subject ship, because the subject ship is far from its regressed region.

5. SIMULATION (WITH MODIFIED HULL COEFFICIENTS)

The first two mathematical models were not sufficient enough to identify accurately the subject ship manoeuvring characteristics. Then employing hull-related coefficients from Lee's [3,4] twin-propeller twin-rudder wide-beam shallow-draft heavy cargo carrier, as well as employing the values described in Section 3 for other propeller- and rudder-related coefficients and the interaction coefficients (it is called "Lee's hull model" in Fig. 6), same simulation was carried out. The results are quite satisfactory comparing to the first simulation sets as shown in the Fig. 6.

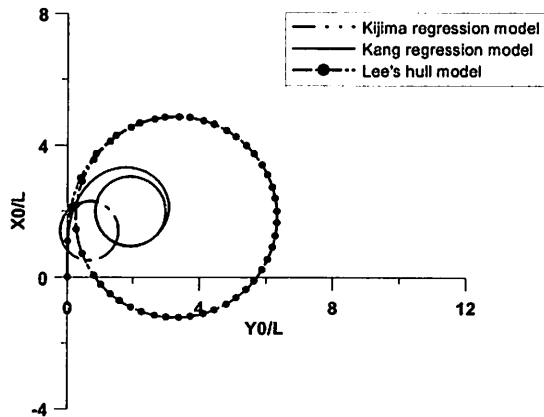


Fig. 6 Simulation of starboard turning ( $\delta \{s, p\} = 15^\circ$ )

It can not be said that the simulation results are accurate enough as there is no precise data to confirm it. At this moment, we can accept Lee's hull coefficients as the best estimation for the subject ship. Finally, as the subject ship mathematical model was roughly estimated, simulations of full scale manoeuvring tests were carried out, assuming constant torque operation of both engines. The simulation results for 25 degrees starboard turning angle are shown in Fig. 7.

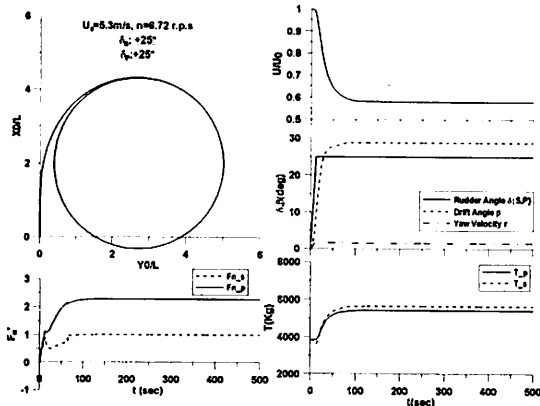


Fig. 7 Simulation of starboard turning ( $\delta \{s, p\} = 25^\circ$ )

$=25^\circ$ )

The simulation shows some reasonable results even for small turning angle. About 32 percent of thrust surge can be also observed in both port and starboard thrust value. Similar phenomenon could be also verified through other researchers experiments (Kobayashi *et al.* [1]) for twin-propeller twin-rudder ship type, even though they did not mention about this fact.

6. COMPARISON OF SIMULATION WITH THE ACTUAL SHIP DATA

The values predicted by the mathematical model developed in the previous section were compared with the full-scale sea trials. Two full scale turning manoeuvres trails respectively for port and starboard turning were carried out by the subject ship operating company in ballast condition and under wind force 3 on Beaufort scale. The simulations were carried out under the same wind conditions during the trials. The wind speed and direction are assumed to be constant, and the effect of the waves is ignored in the simulation.

Figure 7 shows an example of 35 degrees starboard turning simulation corresponding to the full-scale trials. The trajectory of the simulation compares favourably with the tactical diameter (177 m) observed at the full-scale trial. It has to be said here that the exact ship trajectory was not available from the operating company. Only the tactical diameter was measured.

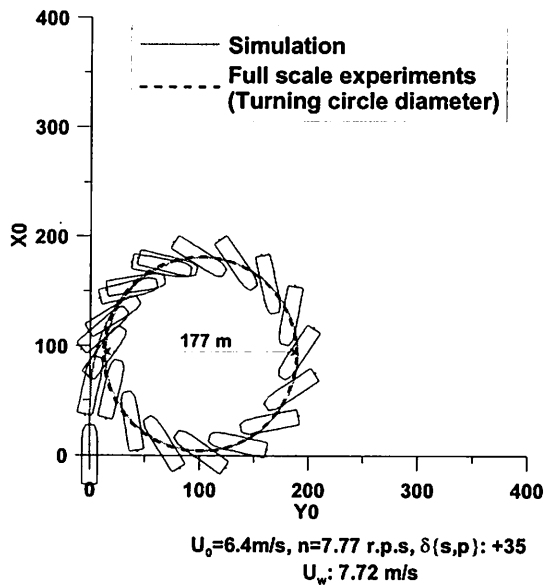


Fig. 8 Simulation of starboard turning

Figure 8 shows the trajectories of the simulation and the actual subject ship sea trials for 35 degrees of port turning. The simulation results for this case don't show a very good agreement as for the previous

case. This can be due to some variations of the wind conditions during the full-scale experiment.

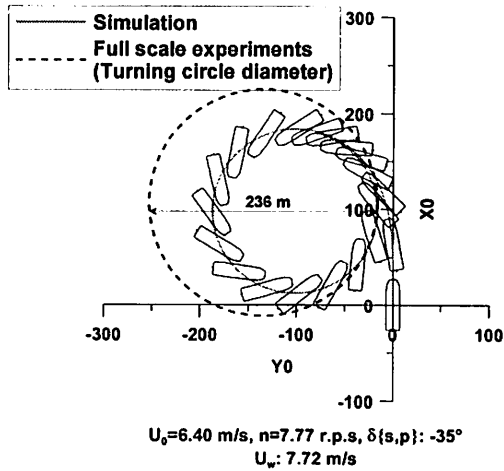


Fig. 9 Simulation of port turning

### 7. MATHEMATICAL MODEL FOR WIND FORCES AND MOMENT

The subject ship is a ferry operating in a regular service of transportation between two harbours distant of about 11 miles in a shallow sea. The ferry presents a series of drawbacks, among which the most important one is the limitation of the navigation by windy days (up to strength 6 Beaufort scale/ 25 knots).

In this study, for numerical simulation, aerodynamic forces induced by steady wind are considered. The expressions of the forces and moment induced by the wind are shown in Eq. (24-26).

$$X_A = \frac{1}{2} \rho_A A_T U_A^2 C_{FX}(\psi_A) \quad (24)$$

$$Y_A = \frac{1}{2} \rho_A A_L U_A^2 C_{FY}(\psi_A) \quad (25)$$

$$M_A = \frac{1}{2} \rho_A A_L U_A^2 C_{MZ}(\psi_A) \quad (26)$$

Where  $C_{FX}$ ,  $C_{FY}$  and  $C_{MZ}$  are aerodynamic drag coefficients.

Fujiwara's regression formula –Fujiwara [11]– is selected to estimate aerodynamic drag coefficient. Fujiwara's regression formula has been developed for new ship types like large size PCCs, LNG carriers, container ships and passenger ships.

In this research, aerodynamic drag coefficients estimated by Fujiwara's equation have been used, and their values for loaded draft condition and for different relative wind angles are shown in Fig. 9 for the subject ship.

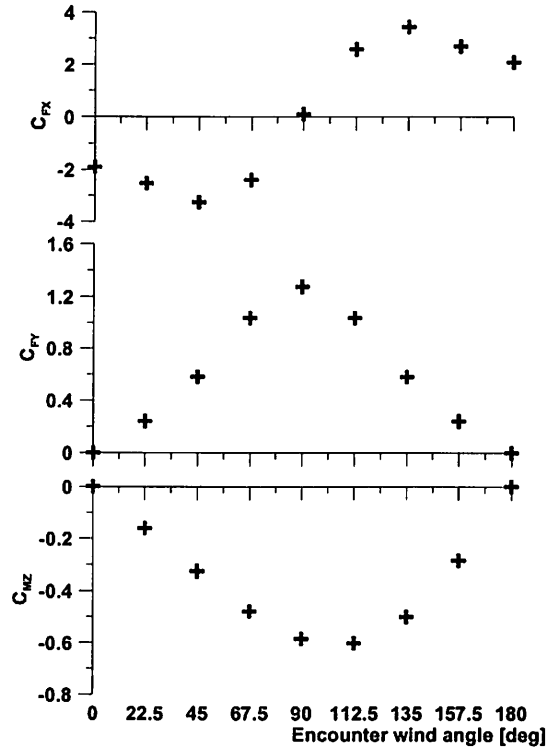


Fig. 9 Coefficients of wind resistance.  $C_{FX}$ ,  $C_{FY}$ , force coefficients in x- and y-direction;  $C_{MZ}$ , coefficient of moment in z-direction.

For this study purpose, ship is simulated in a straight running simulation with certain surge speed  $u$  ( $U_0=6.14\text{m/s}$ ,  $r.p.s=7.92$ ). Then some steady wind is blown corresponding to different wind angles and different wind speeds as it is shown in Fig.10.

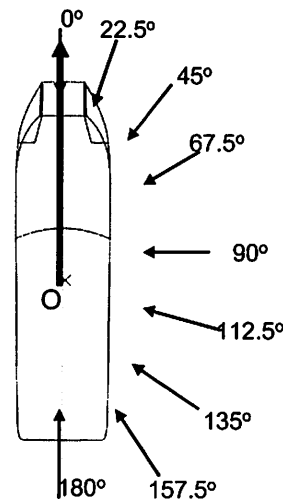


Fig. 10 Wind influence study configuration

The simulations results are plotted respectively for rudder angle, ship sway velocity and ship surge velocity in Figs.11-13.

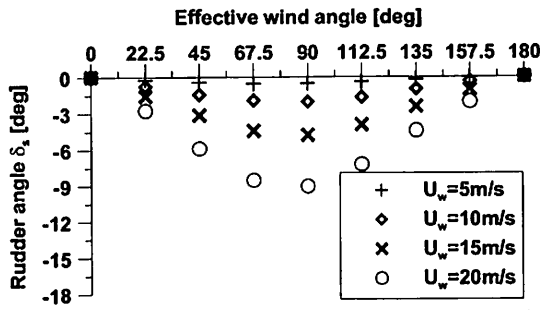


Fig. 11 Wind influence on ship rudder angle ( $\delta_s = \delta_r$ )

The results of different simulations show a deviation of rudders towards the port side in an attempt to countermeasure the yaw turning of ship towards the wind blowing direction from the starboard side. It is also evident from Fig.11 that the maximum rudder response for different wind speeds is at its maximum value for the 90 degrees relative wind angle so called beam wind.

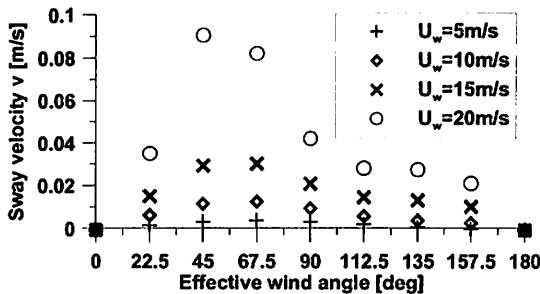


Fig. 12 Wind influence on ship sway velocity

A positive sway velocity with a peak point for a wind velocity of 20 m/s at 45 degrees wind angle is observed in Fig.12. This sway velocity added to the initial surge velocity will induce a drift motion of the ship towards the wind direction.

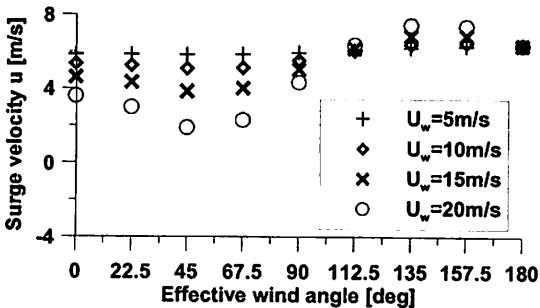


Fig. 13 Wind influence on ship surge velocity

Observing the surge velocity variation in Fig.12, it is easy to figure out that it is a sinusoidal type variation with a minimum value at 45 degrees relative wind angle and a maximum value at 135 degrees wind angle. This can be due to the effect of wind forces and moments acting on the ship superstructure from both sides: the lateral and the transverse superstructure areas. This wind influence on the subject ship is therefore the most significant at 45

degrees and 135 degrees of wind angle. Indeed, it can be notified that for a wind speed of 20 m/s the surge speed drop is about 2/3 from its initial value at 45 degrees wind angle.

Actually the local maritime authority has a regulation limiting navigation on windy days, when the weather condition is higher than strength 6 on Beaufort scale (15m/s). Therefore we demonstrate the practical evidence of the validity of the mathematical model proposed and estimated in this paper.

#### 4. CONCLUSION

In this paper, a method for predicting twin-propeller twin-rudder ship mathematical model coefficients is discussed. Simulation of a twin-propeller twin-rudder ferry under steady wind conditions is carried out. Main conclusions are summarised as follows:

- 1- An approximate mathematical model for the unknown subject ship is totally estimated without conducting any model experiments. A set of combination of hydrodynamic coefficients of a similar hull type ship and the estimated coefficients for propeller, rudder and wind model is established.
- 2- For the subject ship Kijima's regression model seems to be not enough accurate to estimate hull hydrodynamic coefficients of similar ship type as it is far from its regressed region. Kang's regression model shows comparatively better results, while Lee's coefficients give the best results for different manoeuvring simulations. The reason is due to the mother ship data base which consists of mainly slender body ships for the first model and blunt body ships for the second. Lee's subject ship is a wide-beam ship type that is the closest to the subject ship.
- 3- Some typical phenomena specific for the twin-propeller twin-rudder ship such as the thrust surge during turning manoeuvre are observed. However, it is still necessary to investigate further on either twin-propeller twin-rudder ship or wide-beam shallow-draft ship for its hydrodynamic coefficients and on interaction forces and moment of twin-propeller twin-rudder ship.
- 4- The practical regulation for the subject ship navigation limitation established by the local authorities is confirmed as reasonable.

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