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Mathematical Model for Maneuverability and Estimation of Hydrodynamic Coefficients of Twin-Propeller Twin-Rudder Ship

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Key Words: *Twin-propeller twin-rudder ship, Mathematical model of manoeuvrability, Estimation method of hydrodynamic coefficients, Manoeuvring simulation*

1. INTRODUCTION

MMG model is well known as a mathematical model for ship manoeuvrability. Although it is originally designated for single-propeller single-rudder ships, some researchers have attempted and succeeded to expand it for other types of ships. However, it is still very hard to estimate the hydrodynamic coefficients, especially interaction coefficients, without conducting model experiments. Furthermore, conventional empirical formulae to estimate them such as Kijima's regression model are not properly suitable for non-conventional ships, because of lack of sufficient mother data.

In this research the manoeuvring characteristics of a twin-propeller twin-rudder ship were firstly reviewed in details. Moreover, as no experimental data concerning the subject ship were available, a method for estimating the coefficients is proposed in order to identify the mathematical model of the subject ship referring to the manoeuvring study of other multi-propeller multi-rudder model ships developed earlier by other researchers.

Finally, the predicted manoeuvring performance of the subject ship is discussed.

2. TWIN-PROPELLER TWIN-RUDDER SHIP MATHEMATICAL MODEL

Three degrees of freedom model consisting of surge sway and yaw motion is considered. The coordinate system for the twin propeller twin rudder ship is shown in Fig. 1 .

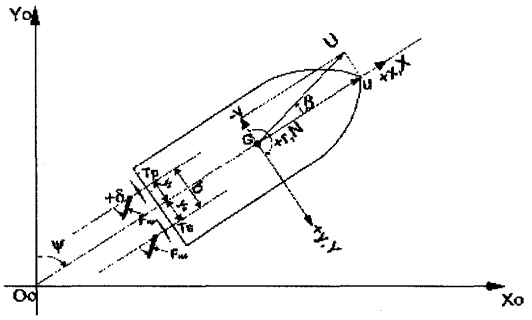

Fig.1 Coordinate system

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The equations of surge, sway and yaw motion considering the origin of coordinate system at the ship center of gravity can be basically written as follows in equation (1):

$$
\begin{aligned}
(m+m_x)\dot{u}-(m+m_y)vr &= X_H+X_P+X_R\\
(m+m_y)\dot{v}+(m+m_x)ur &= Y_H+Y_P+Y_R\\
(I_{zz}+J_{zz})\dot{r} &= N_H+N_P+N_R
\end{aligned} \tag{1}
$$

On the right hand side the subscripts H, P and R stand for force and moment due to hull, propeller and rudder respectively.

Different researches were done and different models were proposed for twin-propeller twin-rudder ships among which the following ones were analyzed:

2. 1 Kobayashi's model[1]:

$$
\begin{aligned}
X_H &= X_{uu}u^2+X_{vv}v^2+X_{vvvv}v^4+X_{rr}r^2+X_{vr}vr\\
Y_H &= Y_v v+Y_{vvv}v^3+Y_r r+Y_{rrr}r^3+Y_{vvr}v^2r+Y_{vrr}vr^2\\
N_H &= N_v v+N_{vvv}v^3+N_r r+N_{rrr}r^3+N_{vvr}v^2r\\
&\quad +N_{vrr}vr^2
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
X_P &= (1-t)(T_{(s)}+T_{(p)})\\
Y_P &= \Delta Y_P=\Delta Y_{P(s)}(J_s)-\Delta Y_{P(p)}(J_s)\\
N_P &= (1-t)\frac{b_P}{2}(T_{(s)}-T_{(p)})\\
&\quad +(\Delta N_{P(p)}(J_s)+\Delta N_{P(p)}(J_s))
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
X_R &= -\left\{(1-t_R)(F_{N(s)}\sin\delta_{(s)}+F_{N(p)}\sin\delta_{(p)})\right\}\\
Y_R &= -\left\{(1+a_H)(F_{N(s)}\cos\delta_{(s)}+F_{N(p)}\cos\delta_{(p)})\right\}\\
N_R &= -\left\{(x_R+a_Hx_H)(F_{N(s)}\cos\delta_{(s)}+F_{N(p)}\cos\delta_{(p)})\right\}\\
&\quad -\frac{b}{2}(1-t_R)(F_{N(p)}\sin\delta_{(p)}-F_{N(s)}\sin\delta_{(s)})
\end{aligned} \tag{4}
$$

The subscripts *H, P, R, N, p* and *s* stand for (force or moment due to) hull, propeller, rudder, port and starboard respectively throughout in this paper.

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The terms ΔY_P, and ΔN_P correspond respectively to the resultant sway force and the added yaw moment due to the interaction forces between both propellers.

2. 2 Yoshimura's model[2]:

$$
\begin{aligned}
X_H &= X0 + X_{\beta\beta}\beta^2 + X_{\beta r}\beta r + X_{rr}r^2 \\
&\quad + X_{\beta\beta\beta r}\beta^3 r + X_{\beta\beta\beta\beta}\beta^4 \\
Y_H &= Y_\beta \beta + Y_r r + Y_{\beta\beta\beta}\beta^3 + Y_{\beta\beta r}\beta^2 r + Y_{\beta rr}\beta r^2 \\
N_H &= N_\beta \beta + N_r r + N_{\beta\beta\beta}\beta^3 + N_{\beta\beta r}\beta^2 r \\
&\quad + N_{\beta rr}\beta r^2 + N_{rrr}r^3
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
X_P &= (1-t)(T_{(s)} + T_{(p)}) \\
N_P &= (1-t)\frac{b_P}{2}(T_{(p)} - T_{(s)})
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
X_R &= -\{1 - t_R - (1-t)C_{\tau\delta}\} \\
&\quad (F_{N(s)}\sin\delta_{(s)} + F_{N(p)}\sin\delta_{(p)}) \\
Y_R &= -\{(1+a_H)(F_{N(s)}\cos\delta_{(s)} + F_{N(p)}\cos\delta_{(p)})\} \\
N_R &= -\{(x_R + a_H x_H)(F_{N(s)}\cos\delta_{(s)} + F_{N(p)}\cos\delta_{(p)})\} \\
&\quad -\frac{b}{2}\{1 - t_R - (1-t)C_{\tau\delta}\}(F_{N(p)}\sin\delta_{(p)} - F_{N(s)}\sin\delta_{(s)})
\end{aligned}
\tag{7}
$$

The term $C_{\tau\delta}$ corresponds to the moment caused by the difference of the longitudinal component of rudder force.

2. 3 Lee's model[3, 4]:

In this model, the origin of coordinate system is set at the center of gravity. The ship equation of motion is therefore written as follows

$$
\begin{aligned}
&(m+m_x)\dot{u} - (m+m_y)vr - mx_G r^2 = X_H + X_P + X_R \\
&(m+m_y)\dot{v} + (mx_G - Y_{\dot{r}})\dot{r} + (m+m_x)ur = \\
&\qquad Y_H + Y_P + Y_R \\
&(I_{zz}+J_{zz})\dot{r} + (mx_G - N_{\dot{v}})\dot{v} + mx_G ur = \\
&\qquad N_H + N_P + N_R
\end{aligned}
\tag{8}
$$

The hydrodynamic forces and moment acting on hull, propeller and rudder are written as in Eqs. (9)-(11) respectively.

$$
\begin{aligned}
X_H &= X_{vv}v^2 + X_{vvvv}v^4 + X_{vr}vr + X_{rr}r^2 \\
Y_H &= Y_v v + Y_r r + Y_{vvv}v^3 + Y_{vvr}v^2 r + Y_{vrr}vr^2 + Y_{rrr}r^3 \\
N_H &= N_v v + N_r r + N_{vvv}v^3 + N_{vvr}v^2 r + N_{vrr}vr^2 \\
&\quad + N_{rrr}r^3
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
X_P &= (1-t)(T_{(s)} + T_{(p)}) \\
N_P &= (1-t)\frac{b_P}{2}(T_{(p)} - T_{(s)})
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
X_R &= -\{(1-t_R)(F_{N(s)}\sin\delta_{(s)} + F_{N(p)}\sin\delta_{(p)})\} \\
Y_R &= -\{(1+a_H)(F_{N(s)}\cos\delta_{(s)} + F_{N(p)}\cos\delta_{(p)})\} \\
N_R &= -\{(x_R + a_H x_H)(F_{N(s)}\cos\delta_{(s)} + F_{N(p)}\cos\delta_{(p)})\} \\
&\quad -\frac{b}{2}(1-t_R)(F_{N(p)}\sin\delta_{(p)} - F_{N(s)}\sin\delta_{(s)})
\end{aligned}
\tag{11}
$$

In this study, Lee's model was chosen as the basic model for the subject ship. The reason is that all coefficients concerning the model were available as well as a proposed regression equation for calculating propeller wake during manoeuvring.

$$
\begin{aligned}
&(1 - w_{P\left\{\substack{s\\p}\right\}}) = (1 - w_{P0}) \\
&+\tau_{\left\{\substack{s\\p}\right\}}\left\{(v_p' + C_{p\left\{\substack{s\\p}\right\}}v_p'|v_p'|^2) + c_{pv\left\{\substack{s\\p}\right\}}v' + c_{pr\left\{\substack{s\\p}\right\}}r'\right\}
\end{aligned}
\tag{12}
$$

Where

$$
\begin{aligned}
v_p' &= -\sin\beta + x_p' r' \\
x_p' &= \frac{x_p}{L} \\
r' &= \frac{r}{\left(\frac{U}{L}\right)}
\end{aligned}
\tag{13}
$$

U, L, x_p, and wp_0 correspond respectively to ship speed, ship length, location of propeller in x axis direction and propeller wake in ahead motion.

3. PREDICTION METHOD OF SUBJECT SHIP HYDRODYNAMIC COEFFICIENTS

The subject ship is a twin-propeller twin-rudder car ferry whose principle particulars are 49.0 m length, 14.0 m width and 2.65 m draught with a hard chin and a transom stern. It is noted that there is neither experiment nor research for the subject ship or similar ships done or published before.

3. 1 Hull forces

The subject ship mathematical model was based originally on Lee's model. The hull forces were estimated using Kang et al (2007)[5] regression method. The equations expressing these hull forces for surge, sway and yaw motion are written in the following equations:

$$
\begin{aligned}
X_H' &= (\mathrm{ax2}\cdot\sin^2(\beta) + \mathrm{ax4}\cdot\sin^2(2\beta))\cdot\cos(\beta) \\
&\quad + \mathrm{bx1}\cdot\sin(\beta)\cdot r' + \mathrm{bx2}\cdot\sin(2\beta)\cdot r'\cdot\mathrm{sign}(\cos(\beta))
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
Y_H' &= (\mathrm{ay1} + \mathrm{cy1}\cdot r'^2)\cdot\sin(\beta) + \mathrm{ay3}\cdot\sin(3\beta) \\
&\quad + \mathrm{ay5}\cdot\sin(5\beta) + (\mathrm{dy1}\cdot r' + \mathrm{ey1}\cdot r'^3)\cdot\cos(\beta)
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
N_H' &= (\mathrm{an2} + \mathrm{cn2}\cdot r'^2)\cdot\sin(2\beta) + \mathrm{an4}\cdot\sin(4\beta) \\
&\quad + \mathrm{dn0}\cdot r' + \mathrm{en0}\cdot r'^3 + \mathrm{dn2}\cdot r'\cdot\cos(2\beta)
\end{aligned}
\tag{16}
$$

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3. 2 Added mass

The added mass and moment of inertia were estimated by Motora et al (1959-1960)[6] diagrams. The moment of inertia I_{ZZ} was estimated as is written in Eqs. (17) and (18).

$$K_{ZZ} = -0.2536.L \quad (17)$$

$$I_{ZZ} = m.K_{ZZ}^{2} \quad (18)$$

Where K_{ZZ} corresponds to the radius of gyration.

3. 3 Ship resistance performance

The subject ship resistance performance was estimated as follows Eq. (19).

$$R_T = 0.5\rho U^2 S\{(1+k)C_{F0} + C_W + C_{BR} + \Delta C_F \quad (19)$$

Where U: ship velocity, S: wetted surface area, k: form factor, C_{F0}: frictional coefficient, C_W: wave making resistance, C_{BR}: shaft brackets effect coefficient, ΔC_F: skin roughness coefficient.

Both coefficients values k and ΔC_F were estimated respectively as 0.15 and 0.005 (Toda, 2008)[7]. The frictional coefficient C_{F0} was calculated according to the ITTC chart (Lewis 1988)[8] as shown in Eq. (20).

$$C_{F_0} = \frac{0.075}{(\log_{10} Rn - 2)^2} \quad (20)$$

Where Rn: Reynolds number.

The wave making resistance C_W and the shaft brackets effect coefficient C_{BR} were determined from the experimental data of similar ship hull type and then they were tuned to fit the subject ship principal particulars dimensions.

3. 4 Propellers and rudders coefficients

The coefficients of thrust deduction fraction t_P and wake fraction w_{P0} were not available and then needed to be estimated as written in Table 1 (Toda, 2008)[7].

The coefficients of propeller thrust in open water were also not available and were estimated by selecting a Wageningen - B series propeller (Kuiper 1992)[9] whose principal characteristics (number of blades, pitch ratio and expanded area ratio) were similar to the propeller fitted on the subject ship, for the operating range of propeller at cruising and maneuvering speed.

Table.1 Propeller coefficients

t_p	0.2
ω_{po}	0.1
C1	0.3721
C2	-0.3142
C3	-0.1521

The rudder drag coefficient was estimated as tR= 0.3 and the coefficients of interaction force between rudder and ship hull were calculated based on a method proposed by Lee et al (1988,2003)3,4 as follows

$$a_H = 0.35$$
$$x_H = -0.4*\mathrm{L} \quad (21)$$

The rudder normal force is expressed as shown in Eq. (22).

$$F_N = \frac{\rho}{2} A_R U_R^{\,2} f_\alpha \cdot \sin\alpha_R \quad (22)$$

Where A_R : rudder area, U_R: Inflow velocity to the rudder, and α_R: effective inflow angle to the rudder.

4. SIMULATION (FIRST TRIAL)

After determining all the hydrodynamic coefficients, simulation of full scale turning tests was carried out and some results are shown in Figs. 2 and 3.

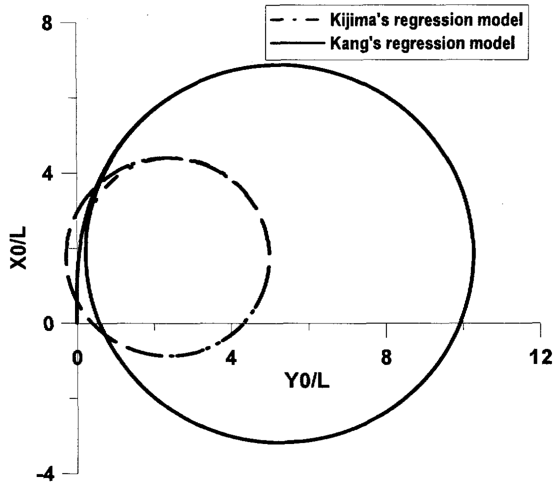

Fig. 2 .Simulation of starboard turning
(Stbd. rudder: 5deg, Port rudder: 5deg)

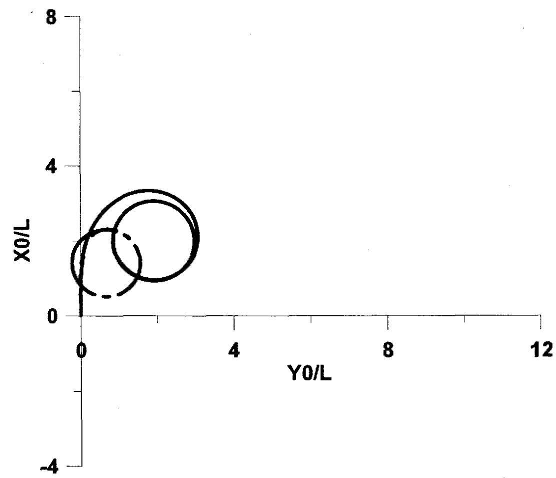

Fig. 3 .Simulation of starboard turning
(Stbd. rudder: 15deg, Port rudder: 15deg)

A first set of simulation was done based on Kang's regression model (Kang et al (2007))[5] for estimating the hydrodynamic forces acting on the hull. The estimated hydrodynamic hull forces showed quite reasonable and acceptable results for large turning angle as it is shown in Fig. 2. However, the more the rudder angle is increased, the more the results seem to be not appropriate and completely irrelevant for 15 degrees turning as shown in Fig. 3.

A second set of simulation was done based on Kijima's regression model (Kijima et al (2003)[10]) for hull

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hydrodynamic forces. It can be said that it is inappropriate for the subject ship, because the subject ship is far from its regressed region.

5. SIMULATION (WITH MODIFIED HULL COEFFICIENTS)

The first two mathematical models were not sufficient enough to identify accurately the subject ship manoeuvring characteristics. Then employing hull-related coefficients from Lee's (1988,2003)[3,4] twin-propeller twin-rudder wide-beam shallow-draft heavy cargo carrier, as well as employing the values described in Section **3** for other propeller- and rudder-related coefficients and the interaction coefficients (it is called "Lee's hull model" in Fig. 4), same simulation was carried out. The results are quite satisfactory comparing to the first simulation sets as shown in the Fig. 4.

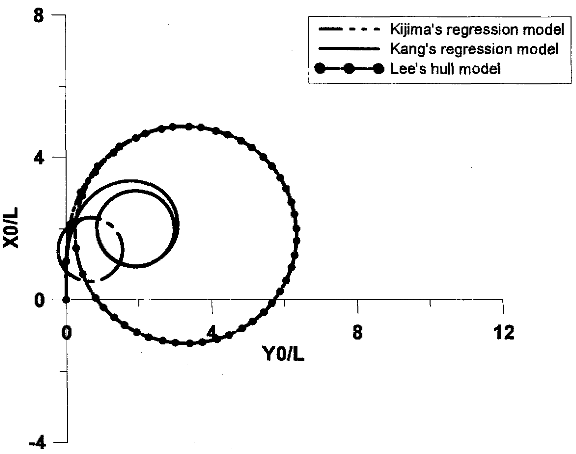

Fig. 4 Simulation of 15 deg starboard turning

Finally, as the subject ship mathematical model was roughly identified, simulations of full scale maneuvering tests were carried out, assuming constant torque operation of both engines. The simulation results for small turning angle are shown in Fig. 5.

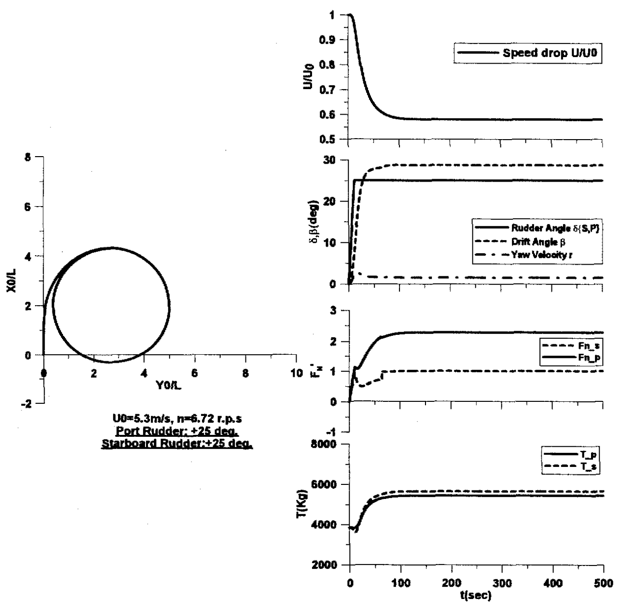

Fig. 5 .Simulation of 25 deg starboard turning

7. CONCLUSIONS

It can be said as a conclusion of this study that Kijima's regression model is not enough accurate to estimate hull hydrodynamic coefficients for the subject ship. Kang's regression model is comparatively better, while Lee's coefficients give the best results for different maneuvering simulations.

These results are due to the mother ship data base which consists of mainly slender body ships for the first model and blunt body ships for the second. Lee's subject ship was a wide-beam ship type that is the closest to the subject ship.

An approximate mathematical model for the unknown subject ship was totally estimated without conducting any model experiments.

Some typical phenomena specific for the twin-propeller twin-rudder ship such as the thrust surge during turning manoeuvre as shown in Fig. 4 were identified.

However, it is still necessary to investigate further on either twin-propeller twin-rudder ship or wide-beam shallow-draft ship for its hydrodynamic coefficients and on interaction forces and moment of twin-propeller twin-rudder ship.

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