Mathematical Model of Manoeuvrability at Low Advance Speed and its Application to Berthing Control

Kazuhiro Hasegawa  
Osaka University  
Suita, Osaka, Japan

Keiji Kitera *  
IBM, Japan  
Yamato, Kanagawa, Japan

Keywords: Manoeuvrability, Mathematical Model, Low Advance Speed, Berthing Control

ABSTRACT

Mathematical model of ship manoeuvrability with current and wind is derived based on MMG-type model. Then the various models at low advance speed are reviewed with recent applications. Combining these models, ship motion at low advance speed with current and wind can be calculated.

As an application of the model automatic berthing control is dealt with. The three layer neural network is designed for this control with a few supervising rules. The system works well not only for the given patterns, but also for some variation of the given patterns including for cases with wind disturbances.

INTRODUCTION

Safety navigation at harbour or congested waterways is of urgent concern. Once a casualty such as collision or being aground happens in this area, the social influence is recently much more than ship, cargo and even man damage, as far as financial basis.

The problems for a ship navigating in these area may be divided into the following summarized points.

- The traffic is heavy in these area.
- Her manoeuvrability at low advance speed is inferior to it at moderate or high speed.
- Environmental disturbances such as wind and current have relatively large effect on her navigation

The first author and others has engaged in the research concerning to the first point. That is, namely the collision avoidance or automatic navigation system using fuzzy theory and expert system (e.g.[1]).

The second point is important as well as the manoeuvrability in shallow water depth. In Japan, a voluntary joint research project named MSS (research project on Manoeuvrability at Slow advance speed and in Shallow water) was started in 1985 following the well-known research project MMG[2, 3]. The aim of the project is to investigate the mathematical model for these conditions from theoretical and/or experimental approaches. The project has finished with its reports[4], and some models were introduced in its third and fourth reports.

However, as the experiments in these conditions are generally difficult than in normal condition, the results were not enough to confirm which model is most recommendable.

In this paper, the mathematical model used in the previous paper[1] is first introduced as a model dealing with non-uniform current and wind, and then a mathematical model at slow advance speed proposed by Kose et al.[20] is introduced, referring some other works.

Finally, the latter model is applied for berthing control using neural network. Both the model and the control method were confirmed promising.

MATHEMATICAL MODEL WITH CURRENT AND WIND

MMG-type model of ship motions in current and wind

MMG model[2, 3] is now widely used as a mathematical model of ship manoeuvring motions at moderate or high ship speed, though there are minor variations in its implementation.

The following set of equations is one of these expressions according to the coordinate system shown in Figure 1.

\[
(m + m_x)\ddot{u} = (m + m_y + X_w)\nu_{ca} r + X_{uv} x_{ca}^2 + X_{rr} r^2 - \left(m + m_x\right)U_c r \sin(\psi_c - \psi) + (1 - t(t - R)^2 - (1 - t)F_N \sin \delta + X_w
\]

\[
(m + m_y)\ddot{v} - Y_r \ddot{r} = Y_{vca} + (Y_{v - m u_{ca}}) r + Y_{r-N}(\nu_{ca} r) + (m + m_y)U_c r \cos(\psi_c - \psi) - (1 + a_H)F_N \cos \delta + Y_w
\]

\[
(I_{zz} + J_{zz}) \ddot{\phi} - N_0\ddot{v} = N_0 \nu_{ca} + N_r + N_{NL}(\nu_{ca} r) - N_u U_c \cos(\psi_c - \psi) - (x_L + a_H x_H)F_N \cos \delta + N_w
\]

where longitudinal and transverse relative flow velocities
by current are treated in a similar manner as \[5, 6\]

\begin{align}
\dot{u}_{ca} &= u + U_c \cos(\psi_c - \psi) \\
\dot{v}_{ca} &= v + U_c \sin(\psi_c - \psi) \\
\dot{w}_{ca} &= \dot{u} + U_c \sin(\psi_c - \psi) \\
\dot{w}_{ca} &= \dot{v} - U_c \cos(\psi_c - \psi)
\end{align}

There are various expressions concerning to the nonlinear hydrodynamic force (transverse) and moment (around z-axis), but the following cross-flow model is effective for treating non-uniform current (refer [8] for more detail).

\begin{align}
Y'_{NL} &\equiv Y_{NL}/\frac{\rho U_c^2 L d}{2} \\
&= -C_D \int_{-1/2}^{1/2} [v'_{ca} + \xi r'] (v'_{ca} + \xi r') d\xi \\
N'_{NL} &\equiv N_{NL}/\frac{\rho U_c^2 L^2 d}{2} \\
&= -C_D \int_{-1/2}^{1/2} [v'_{ca} + \xi r'] (v'_{ca} + \xi r') \xi d\xi
\end{align}

where

\begin{align}
v'_{ca} &= v_{ca}/U_c \\
r' &= r/L/U_c \\
\xi &= x/L \\
U_c &= \sqrt{u_{ca}^2 + v_{ca}^2}
\end{align}

\begin{align}
N'_{NL} &= N_{NL}/\frac{\rho U_c^2 L^2 d}{2} \\
N'_{NL} &= -C_D \int_{-1/2}^{1/2} [v'_{ca} + \xi r'] (v'_{ca} + \xi r') \xi d\xi
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\xi &= x/L \\
U_c &= \sqrt{u_{ca}^2 + v_{ca}^2}
\end{align}

\begin{align}
Y'_{NL} &= -\left(\frac{\pi}{2} \Lambda + 1.4 C_B \frac{B}{L} \right) \left(1 + \frac{2 \tau}{3 d_m}\right) \\
Y'_{r} &= \frac{\pi}{4} \Lambda \left(1 + 0.8 \frac{\tau}{d_m}\right) \\
N'_{v} &= -\Lambda \left(1 - \frac{0.27 \tau}{l_B d_m}\right) \\
N'_{r} &= -\left(0.54 \Lambda - \Lambda^2\right) \left(1 + 0.3 \frac{\tau}{d_m}\right)
\end{align}

where

\begin{align}
\Lambda &= \frac{2d_m}{L} \\
l_B &= \Lambda/(-\pi \Lambda + 1.4 C_B \frac{B}{L})
\end{align}

Added mass and added moment of inertia

Motora's charts[8] are very reliable, or it is also convenient to use the regression forms proposed by Clarke et al[3].

\begin{align}
m'_{v} &= m_{v}/\frac{\rho L^2 d}{2} \\
&= \frac{\pi d}{L} \left(1 + 0.16 C_B \frac{B}{d} - 5.1 \left(\frac{B}{L}\right)^2\right) \\
J'_{z} &= J_{zz}/\frac{\rho L^4 d}{2} \\
&= \frac{\pi d}{L} \left[\frac{1}{12} + 0.017 C_B \frac{B}{d} - 0.33 \left(\frac{B}{L}\right)^2\right] \\
Y'_{r} &= Y_{r}/\frac{\rho L^2 d}{2} \\
&= -\frac{\pi}{L} \left(0.67 \frac{B}{L} - 0.033 \left(\frac{B}{d}\right)^2\right) \\
N'_{r} &= N_{r}/\frac{\rho L^3 d}{2} \\
&= -\frac{d}{L} \left(1.1 \frac{B}{L} - 0.041 \frac{B}{d}\right)
\end{align}

Nonlinear derivatives and cross-flow drag coefficient

Inoue et al. have also proposed the charts for them[7] and Kijima et al. have obtained the regression forms of them[10].

Cross-flow drag coefficient can be assumed as

\begin{align}
C_D &= -Y'_{v}
\end{align}

Rudder coefficients

If data of rudder is available, so-called MMG's model[2, 3] procedure to estimate natural rudder force coefficient, effective rudder inflow velocity and angle is usable.

Dynamics of steering gear

Dynamics of a steering gear is well modelled by the following equations.

\begin{align}
T_E \dot{\delta} + \delta &= \delta^* \\
\dot{\delta} &\leq \dot{\delta}_{\text{max}}
\end{align}
Normally the regulations require to take 28 sec for 65 deg steering, so $T_0$ is around 2.5 sec, and the capacity of average electro-hydraulic pump provides $\delta_{\text{max}}$ as around 3.5 deg/sec.

**Aerodynamic forces and moment**

Wind forces and moment acting on a ship can be expressed as follows according to Isherwood's formula [11].

\[
X_w = C_X \frac{1}{2} \rho_w U_{wa}^2 A_T
\]

\[
Y_w = C_Y \frac{1}{2} \rho_w U_{wa}^2 A_L
\]

\[
N_w = C_N \frac{1}{2} \rho_w U_{wa}^2 r A_L L_{OA}
\]

where

\[
U_{wa} = \sqrt{u_{wa}^2 + v_{wa}^2}
\]

\[
\psi_{wa} = \arctan \frac{v_{wa}}{u_{wa}}
\]

\[
u_{wa} = u + U_w \cos(\psi - \psi_{wa})
\]

\[
v_{wa} = u + U_w \sin(\psi - \psi_{wa})
\]

and

\[
C_X = A_0 + A_1 \frac{2A_L}{L_{OA}} + A_2 \frac{2A_T}{B^2}
\]

\[
+ A_3 \frac{L_{OA}}{B} + A_4 \frac{SI}{L_{OA}} + A_5 \frac{C}{L_{OA}}
\]

\[
+ A_6 M
\]

\[
C_Y = B_0 + B_1 \frac{2A_L}{L_{OA}} + B_2 \frac{2A_T}{B^2}
\]

\[
+ B_3 \frac{L_{OA}}{B} + B_4 \frac{SI}{L_{OA}} + B_5 \frac{C}{L_{OA}}
\]

\[
+ B_6 \frac{ASS}{A_L}
\]

\[
C_N = C_0 + C_1 \frac{2A_L}{L_{OA}} + C_2 \frac{2A_T}{B^2}
\]

\[
+ C_3 \frac{L_{OA}}{B} + C_4 \frac{SI}{L_{OA}} + C_5 \frac{C}{L_{OA}}
\]

(1) **Polynomial Models**

If the continuity between MMG model is concerned, polynomial models are superior, though there are several ideas.

**Hamamoto–Hasegawa model [13]**

The continuity between MMG model and expansion for large amplitude of sway velocity are treated.

H. Kobayashi[14] for tug operation

**E. Kobayashi–Asai model [15]**

Four models are connected smoothly according to the advance speed.

E. Kobayashi[16] for various simulation at low advance speed

**Takashina model [17]**

The lateral force and moment with large sway velocity are expressed in Fourier expansion and converted into polynomial terms accounting major terms.

Hirano[18] for tug operation

Sohn[19] for various simulation at low advance speed

**Kose model [20]**

Standing the point that the ship speed $U$ cannot be used for the non-dimensionalization of forces and moment, $g$ is adopted instead. The continuity to MMG model should be studied.

Yamato, Komai et al.[21] for berthing control

Fujino[22] for stopping manoeuvres

**Shoji model [23]**

Rewritten the forces and moment avoiding to divide by the ship speed $U$.

Shoji, Ohata and Mizoguchi[24] for berthing control

(2) **Fourier Expansion Models**

The Fourier expansion model is very natural from the experiment results of oblique towing tests. The forces and moment are mostly expressed in Fourier series. The problem is how to deal with yaw terms. It is also known this model is well expressed in polynomial model.

**Takashina model [17]**

Treating the yaw terms in 2nd order polynomials

**Yumuro model [25]**

Similar to Takashina model but employing only sine terms

**Khattab model [26]**

Treating rate of turn in effective sway velocity
(3) Cross-Flow Models

Cross-Flow model is based on the cross flow drag acting on a ship. However, the distribution of cross flow drag alongside a ship is hardly obtained or estimated, more precise estimation and at the same time simpler models are required.

Oltmann model [27]

Dividing the hydrodynamic force and moment into some physical meaningful terms

Karasuno model [28]

Similar to Oltmann model, but simplified it

Yoshimura model [29]

Treating $C_D$ as a constant alongside a ship

(4) Response Equation Model

Nomoto’s $K-T$ model is still widely used to predict ship manoeuvrability, especially at full-scale trials. The model is expanded to 2nd order $K-T$ model and its nonlinear model for VLCC/ULCC’s. This model is also expandable at low advance speed.

Yoshimura-Nomoto model [30]

This model is maybe the first model treating various advance speed with various propeller revolution. The model is very simple and easily implemented into an analog computer, so it is used for first generation ship handling simulators.

Aiguo[31] for ship handling simulator

In this study, we adopted several polynomial models for simulation program, but here Kosw model will be briefly introduced in the next subsection.

Hydrodynamic forces and moment acting on a hull at low advance speed

Non-dimension convention

Kose et al.[29] proposed the non-dimension factor for force as $\rho/2\cdot L^3$ instead of $\rho/2\cdot L^3\cdot U$. This is because to avoid the divergence on terms divided by $U$, when the ship advance speed is zero. The other physical terms are non-dimensionalized in the following ways.

\[
X^*, Y^* = \frac{X, Y}{\frac{\rho}{2} \cdot L^3} \quad (40)
\]

\[
N^* = \frac{N}{\frac{\rho}{2} \cdot L^4} \quad (41)
\]

\[
m^* = \frac{m}{\frac{\rho}{2} \cdot L^5} \quad (42)
\]

\[
I_{rr}^* = \frac{I_{rr}}{\frac{\rho}{2} \cdot L^5} \quad (43)
\]

\[
u^*, v^* = \frac{u, v}{\sqrt{L/g}} \quad (44)
\]

\[
r^* = \frac{r}{\sqrt{L/g}} \quad (45)
\]

\[
a^*, b^* = \frac{a, b}{g} \quad (46)
\]

\[
i^* = \frac{i \cdot L}{g} \quad (47)
\]

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship length</td>
<td>$L$</td>
<td>304.0</td>
<td>m</td>
</tr>
<tr>
<td>Length overall</td>
<td>$L_{OA}$</td>
<td>310.5</td>
<td>m</td>
</tr>
<tr>
<td>Ship breadth</td>
<td>$B$</td>
<td>52.5</td>
<td>m</td>
</tr>
<tr>
<td>Mean draught</td>
<td>$d_m$</td>
<td>17.4</td>
<td>m</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>$C_b$</td>
<td>0.827</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>2.350 x 10^4</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$I_{zz}$</td>
<td>1.018 x 10^{12}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Wetted surface area</td>
<td>$S$</td>
<td>2.259 x 10^4</td>
<td>m^2</td>
</tr>
<tr>
<td>Propeller diameter</td>
<td>$D_p$</td>
<td>8.5</td>
<td>m</td>
</tr>
<tr>
<td>Propeller pitch</td>
<td>$P$</td>
<td>5.16</td>
<td>m</td>
</tr>
<tr>
<td>Rudder area</td>
<td>$A_R$</td>
<td>98.0</td>
<td>m^2</td>
</tr>
<tr>
<td>Rudder height</td>
<td>$h$</td>
<td>12.94</td>
<td>m</td>
</tr>
<tr>
<td>Rudder aspect ratio</td>
<td>$A_R$</td>
<td>1.709</td>
<td></td>
</tr>
</tbody>
</table>

Expression of hull forces and moment

Transverse and lateral forces and moment around z-axis are modelled in the following equations.

\[
X^*_H = X^*_r v^* r^* + X^*_u |u^*| u^* - X^*_w v^* v^2 / U^* + X^*_w r^* / U^* \quad (48)
\]

\[
Y^*_H = Y^*_r U^* v^* + Y^*_u |v^*| u^* + Y^*_w u^* v^* U^* + Y^*_w u^* r^* U^* + Y^*_w v^* r^* / U^* \quad (49)
\]

\[
N^*_H = N^*_w u^* v^* + N^*_r r^* + N^*_u u^* r^* + N^*_w u^* v^* r^* \quad (50)
\]

In the eqs.(48-50), $u$ and $v$ should be replaced with $u_{ca}$ and $v_{ca}$ defined in eqs.(4–5), where current exists.

BERTING CONTROL USING NEURAL NETWORK

Berthing is one of the most difficult manoeuvres in ship operation. A captain or a pilot should take care of the ship’s manoeuvrability, current and wind conditions, harbour configuration, other traffic and tug operations. If all or some part of berthing is automated or guided by a computer, it will be of great help for them.

In this section, an approach for it is shown as an application of mathematical model at low advance speed.

Ship particulars and derivatives

A tanker is considered for the simulation. In Table 1, the principal particulars of the tanker is shown. The constants concerning to the wind resistance are listed on Table 2. Hydrodynamical derivatives concerning to $X_H$, $Y_H$ and $N_H$ are listed on Table 3.

Neural control of berthing

Neural controller was designed for berthing control. Three layer network with error back propagation
Table 2  Wind resistance constants.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_L )</td>
<td>4135.6</td>
<td>m²</td>
</tr>
<tr>
<td>( A_T )</td>
<td>993.5</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{SS} )</td>
<td>1047.5</td>
<td>m²</td>
</tr>
<tr>
<td>( C )</td>
<td>164.5</td>
<td>m</td>
</tr>
<tr>
<td>( S_l )</td>
<td>416.7</td>
<td>m</td>
</tr>
<tr>
<td>( M )</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Table 3  Hydrodynamic derivatives.

| \( X^*_{xu} \) | \(-0.310 \times 10^{-2}\) |
| \( X^*_{yu} \) | \(-0.457 \times 10^{-2}\) |
| \( X^*_{yu} \) | \(2.927 \times 10^{-3}\) |
| \( X^*_{xu} \) | \(-0.558 \times 10^{-3}\) |
| \( Y^*_x \)  | \(-2.222 \times 10^{-2}\) |
| \( Y^*_y \)  | \(-2.173 \times 10^{-2}\) |
| \( Y^*_{uwx} \) | 0.0 |
| \( Y^*_{ux} \) | \(1.287 \times 10^{-5}\) |
| \( Y^*_{uy} \) | \(0.510 \times 10^{-2}\) |
| \( Y^*_{ux} \) | \(-0.851 \times 10^{-2}\) |
| \( Y^*_{uy} \) | \(-0.874 \times 10^{-2}\) |
| \( N^*_{xu} \) | \(-7.910 \times 10^{-3}\) |
| \( N^*_{yu} \) | \(-1.632 \times 10^{-4}\) |
| \( N^*_{xy} \) | \(-0.776 \times 10^{-2}\) |
| \( N^*_{xw} \) | \(-1.584 \times 10^{-3}\) |
| \( N^*_{yw} \) | \(-0.981 \times 10^{-3}\) |

Figure 2  Three layer neural network.

Figure 3  Definition of \( \xi \) and \( \eta \).

method is employed. Similar network was used by Yamamoto et al. [32], but some modification to improve the performance was done. The detail description will be found in [33]. Neural network is designed as shown in Figure 2, where left-hand side neurons are inputs layer, middle column neurons are middle layer and right-hand side neurons are outputs layer. As inputs, 7 variables as shown in the figure are used, where \( \xi \) and eta are defined as shown in Figure 3.

Learning parameters

After providing teaching data, which was taken from the simulation results of manual berthing done by one of the authors, network parameters were determined by learning. Learning procedure is finished by monitoring the root mean square value of errors. Figure 4 shows 7 patterns used for teaching data. Figure 5 is one of the example of teaching data (○ denotes a sampled point used for learning).
Figure 4 Patterns of teaching data.

Figure 5 An example of given teaching data: starting at \((5L, 4L)\) with \(\psi = -40\) deg.

Figure 6 Simulation starting at \((5L, 4L)\) with \(\psi = -40\) deg.

Results of simulation

Simulation of automatic berthing is done with various initial conditions including those of teaching data. In this paper, berthing manoeuvres phase I (defined by Yamato [32]) is treated; i.e. when the ship approaches to the pier inside a given area (here \(L/2\) square) with less than a given velocity (here 0.5 m/sec), the control task is regarded as being finished. At the initial conditions as same as those of teaching data, the results are of course satisfactory. Figure 6 shows a result of the same condition as Figure 5. The trajectory looks almost same as that of teaching data, but in the time history we can notice that the neural controller adjusted the control (of rudder and propeller revolution) so as to follow the teaching trajectory. The neural controller has an advantage when the initial conditions (entering point, angle and velocity) have changed. It is usually difficult to adjust parameters for such conditions in the cases of ordinary control methods. Figure 7 (entering angle is different), Figure 8 (entering position is different) and Figure 9 (entering position is different) verify this. Wind effect is also checked. Figure 10 (heading wind), Figure 11 (lee wind) and Figure 12 (following wind) show the effect of various wind direction. (The needle in the above circle shows the wind direction.) The controller adjusts the amount or the timing of commands according to the wind condition.
Figure 7  Simulation starting at (5L, 4L) with $\psi = -70\text{deg}$.

Figure 9  Simulation starting at (6L, 2L) with $\psi = -30\text{deg}$.

Figure 8  Simulation starting at (5L, 3L) with $\psi = -40\text{deg}$.

Figure 10  Simulation starting at (5L, 3L) with $\psi = -30\text{deg}$ (fair wind).
CONCLUSIONS

Mathematical models of ship manoeuvring at low advance speed are reviewed mainly from Japanese papers. Kose model is chosen from them and applied for berthing control.

The main conclusions obtained are summarized as follows:

1. Mathematical models of ship manoeuvrability at low advance speed are summarized with recent applications.

2. Neural controller designed for berthing control is verified to work satisfactory for various conditions.

The following points are the recommendation for the future study.

1. More researches on mathematical model at low advance speed are required to standardize it.

2. More captive and free-running models tests should be done for various types of ships to gather derivatives for the model.

3. It is required to establish the method to estimate the derivatives of the model from the practical purpose.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to Prof. K. Kose, Hiroshima University and Prof. H. Hinata, Maritime Safety Academy for providing the data of their experiments.

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NOMENCLATURE

Physical constants

\( g \) : gravity acceleration
\( \rho_a \) : density of air
\( \rho \) : density of water

Principal particulars

\( L \) : ship length between perpendiculars
\( L_{OA} \) : ship overall length
\( B \) : ship breadth
\( d_{m} \) : ship mean draught
\( \tau \) : trim
\( \Lambda \) : ship aspect ratio

Mass and added masses

\( m \) : mass of a ship
\( m_L \) : longitudinal added mass of a ship
\( m_T \) : transverse added mass of a ship
\( I_{xx} \) : moment of inertia around \( x \)-axis of a ship
\( J_{zz} \) : added moment of inertia around \( z \)-axis of a ship

Velocities and angles

\( u \) : longitudinal component of ship velocity \( U \)
\( v \) : transverse component of ship velocity \( U \)
\( r \) : rate of turn around \( x \)-axis of a ship
\( \psi \) : ship direction
\( \delta^* \) : rudder command
\( \delta \) : rudder angle

Current parameters

\( U_c \) : current velocity
\( \psi_c \) : current direction
\( u_{rel}, v_{rel} \) : longitudinal and transverse relative flow velocities by current

Wind parameters

\( U_w \) : wind velocity
\( \psi_w \) : wind direction
\( u_{rel}, v_{rel} \) : longitudinal and transverse relative velocities by wind
\( U_{rel} \) : relative velocity by wind
\( \psi_{rel} \) : relative direction of wind

Thrust and resistance

\( T \) : propeller thrust
\( R \) : longitudinal resistance of hull with rudder
\( t \) : thrust deduction coefficient
\( t_R \) : deduction coefficient of rudder nominal force

Hydrodynamic forces

\( Y_{NL}, N_{NL} \) : nonlinear hydrodynamic force (transverse) and moment (around \( z \)-axis)
\( C_D \) : cross-flow drag coefficient

Rudder force

\( F_N \) : nominal force acting on a rudder

Wind resistance coefficients

\( X_w, Y_w, N_w \) : aerodynamic forces (longitudinal and transverse), and moment (around \( z \)-axis) acting on a ship by wind
\( C_X, C_Y, C_N \) : coefficients (longitudinal, transverse forces and moment) of wind resistances
\( A_L \) : lateral projected area of a ship above waterline
\( A_T \) : transverse projected area of a ship above waterline
\( A_{SS} \) : lateral projected area of superstructure
\( S_l \) : length of perimeter of lateral projection of a ship excluding waterline and slender bodies such as masts and ventilators
\( C \) : distance from bow of centroid of lateral projected area
\( M \) : number of distinct groups of masts or kingposts seen in lateral projection; kingposts close against the bridge front are not included
\( A_0 \cdots A_6 \) : regression coefficients of wind resistances[11]

Steering gear parameters

\( T_E \) : time constant of a steering gear (sec)
\( \delta_{max} \) : maximum rudder speed (deg/sec)