Proc. of 2nd Japan-Korea Joint Workshop on Ship and Marine Hydrodynamics, Osaka, June 28-30, 1993.

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Mathematical Model of Manoeuvrability at Low Advance Speed and its Application to Berthing Control

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Keywords; Manoeuvrability, Mathematical Model, Low Advance Speed, Berthing Control

ABSTRACT

Mathematical model of ship manoeuvrability with current and wind is derived based on MMG-type model. Then the various models at low advance speed are reviewed with recent applications. Combining these models, ship motion at low advance speed with current and wind can be calculated.

As an application of the model automatic berthing control is dealt with. The three layer neural network is designed for this control with a few supervising rules. The system works well not only for the given patterns, but also for some variation of the given patterns including for cases with wind disturbances.

INTRODUCTION

Safety navigation at harbour or congested waterways is of urgent concern. Once a casualty such as collision or being aground happens in these area, the social influence is recently much more than ship, cargo and even man damage, as far as financial basis.

The problems for a ship navigating in these area may be divided into the following summarized points.

- The traffic is heavy in these area.
- Her manoeuvrability at low advance speed is inferior to it at moderate or high speed.
- Environmental disturbances such as wind and current have relatively large effect on her navigation

The first author and others has engaged in the research concerning to the first point. That is, namely the collision avoidance or automatic navigation system using fuzzy theory and expert system (*e.g.*[1]).

The second point is important as well as the manoeuvrability in shallow water depth. In Japan, a voluntary joint research project named MSS (research project on Manoeuvrability at Slow advance speed and in Shallow water) was started in 1985 following the well-known research project MMG[2, 3]. The aim of the project is to investigate the mathematical model for these conditions from theoretical and/or experimental approaches. The project has finished with its reports[4], and some models were introduced in its third and fourth reports.

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However, as the experiments in these conditions are generally difficult than in normal condition, the results were not enough to confirm which model is most recommendable.

In this paper, the mathematical model used in the previous paper[1] is first introduced as a model dealing with non-uniform current and wind, and then a mathematical model at slow advance speed proposed by Kose *et al.*[20] is introduced, referring some other works.

Finally, the latter model is applied for berthing control using neural network. Both the model and the control method were confirmed promising.

MATHEMATICAL MODEL WITH CURRENT AND WIND

MMG-type model of ship motions in current and wind

MMG model[2, 3] is now widely used as a mathematical model of ship manoeuvring motions at moderate or high ship speed, though there are minor variations in its implementation.

The following set of equations is one of these expressions according to the coordinate system shown in Figure 1.

$$
\begin{aligned}
(m+m_x)\dot{u} &= (m+m_y+X_{vr})v_{ca}r \\
&\quad +X_{vv}v_{ca}^2+X_{rr}r^2 \\
&\quad -(m+m_x)U_c r\sin(\psi_c-\psi) \\
&\quad +(1-t)T-R \\
&\quad -(1-t_R)F_N\sin\delta \\
&\quad +X_w \qquad (1)\\
(m+m_y)\dot{v}-Y_{\dot{r}}\dot{r} &= Y_v v_{ca}+(Y_r-mu_{ca})r \\
&\quad +Y_{NL}(v_{ca},r) \\
&\quad +(m+m_y)U_c r\cos(\psi_c-\psi) \\
&\quad -(1+a_H)F_N\cos\delta \\
&\quad +Y_w \qquad (2)\\
(I_{zz}+J_{zz})\dot{r}-N_{\dot{v}}\dot{v} &= N_v v_{ca}+N_r r \\
&\quad +N_{NL}(v_{ca},r) \\
&\quad -N_{\dot{v}}U_c r\cos(\psi_c-\psi) \\
&\quad -(x_R+a_H x_H)F_N\cos\delta \\
&\quad +N_w \qquad (3)
\end{aligned}
$$

where longitudinal and transverse relative flow velocities

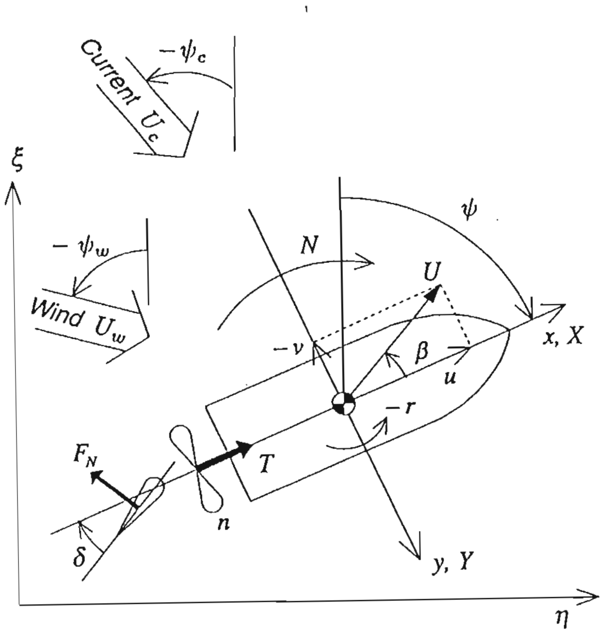

Figure 1 Coordinate system.

by current are treated in a similar manner as [5, 6]

$$
\begin{aligned}
u_{ca} &= u + U_c \cos(\psi_c - \psi) & (4)\\
v_{ca} &= v + U_c \sin(\psi_c - \psi) & (5)\\
\dot{u}_{ca} &\simeq \dot{u} + U_c r \sin(\psi_c - \psi) & (6)\\
\dot{v}_{ca} &\simeq \dot{v} - U_c r \cos(\psi_c - \psi) & (7)
\end{aligned}
$$

There are various expressions concerning to the nonlinear hydrodynamic force (transverse) and moment (around $z-$axis), but the following cross-flow model is effective for treating non-uniform current (refer [6] for more detail).

$$
\begin{aligned}
Y'_{NL} &\equiv Y_{NL}/\frac{\rho}{2}U_{ca}^2 Ld \\
&= -C_D \int_{-1/2}^{1/2} |v'_{ca} + \xi r'|(v'_{ca} + \xi r')d\xi & (8)\\
N'_{NL} &\equiv N_{NL}/\frac{\rho}{2}U_{ca}^2 L^2 d \\
&= -C_D \int_{-1/2}^{1/2} |v'_{ca} + \xi r'|(v'_{ca} + \xi r')\xi d\xi & (9)
\end{aligned}
$$

where

$$
\begin{aligned}
v'_{ca} &= v_{ca}/U_{ca} & (10)\\
r' &= rL/U_{ca} & (11)\\
\xi &= x/L & (12)\\
U_{ca} &= \sqrt{u_{ca}^2 + v_{ca}^2} & (13)
\end{aligned}
$$

Estimation of derivatives

If data of captive model tests etc. are not available, following formulae are useful to estimate the derivatives.

Linear derivatives

Inoue *et al.* have derived the following semi-empirical formulae[7].

$$
\begin{aligned}
Y'_v &= -(\frac{\pi}{2}\Lambda + 1.4C_B\frac{B}{L})(1 + \frac{2}{3}\frac{\tau}{d_m}) & (14)\\
Y'_r &= \frac{\pi}{4}\Lambda(1 + 0.80\frac{\tau}{d_m}) & (15)\\
N'_v &= -\Lambda(1 - \frac{0.27}{l_\beta}\frac{\tau}{d_m}) & (16)\\
N'_r &= -(0.54\Lambda - \Lambda^2)(1 + 0.30\frac{\tau}{d_m}) & (17)
\end{aligned}
$$

where

$$
\begin{aligned}
\Lambda &= \frac{2d_m}{L} & (18)\\
l_\beta &= \Lambda/(-\pi\Lambda + 1.4C_B\frac{B}{L}) & (19)
\end{aligned}
$$

Added mass and added moment of inertia

Motora's charts[8] are very reliable, or it is also convenient to use the regression forms proposed by Clarke *et al.*[9].

$$
\begin{aligned}
m'_y &\equiv m_y/\frac{\rho}{2}L^2 d \\
&= \pi\frac{d}{L}\{1 + 0.16C_B\frac{B}{d} - 5.1(\frac{B}{L})^2\} & (20)\\
J'_{zz} &\equiv J_{zz}/\frac{\rho}{2}L^4 d \\
&= \pi\frac{d}{L}\{\frac{1}{12} + 0.017C_B\frac{B}{d} - 0.33\frac{B}{L}\} & (21)\\
Y'_{\dot{r}} &\equiv Y_{\dot{r}}/\frac{\rho}{2}L^3 d \\
&= -\pi\frac{d}{L}\{0.67\frac{B}{L} - 0.033(\frac{B}{d})^2\} & (22)\\
N'_{\dot{v}} &\equiv N_{\dot{v}}/\frac{\rho}{2}L^3 d \\
&= -\pi\frac{d}{L}\{1.1\frac{B}{L} - 0.041\frac{B}{d}\} & (23)
\end{aligned}
$$

Nonlinear derivatives and cross-flow drag coefficient

Inoue *et al.* have also proposed the charts for them[7] and Kijima *et al.* have obtained the regression forms of them[10].

Cross-flow drag coefficient can be assumed as

$$
C_D = -Y'_{vv} \qquad (24)
$$

Rudder coefficients

If data of rudder is available, so-called MMG's model[2, 3] procedure to estimate normal rudder force coefficient, effective rudder inflow velocity and angle is usable.

Dynamics of steering gear

Dynamics of a steering gear is well modelled by the following equations.

$$
\begin{aligned}
T_E\dot{\delta} + \delta &= \delta^* & (25)\\
\dot{\delta} &\leq \dot{\delta}_{\max} & (26)
\end{aligned}
$$

Normally the regulation requires to take 28 sec for 65 deg steering, so T_E is around 2.5 sec. and the capacity of average electro-hydraulic pump provides $\dot{\delta}_{\max}$ as around 3.5 deg/sec.

Aerodynamic forces and moment

Wind forces and moment acting on a ship can be expressed as follows according to Isherwood's formulae[11].

$$X_w = C_X \frac{1}{2}\rho_a U_{wa}^2 A_T \tag{27}$$

$$Y_w = C_Y \frac{1}{2}\rho_a U_{wa}^2 A_L \tag{28}$$

$$N_w = C_N \frac{1}{2}\rho_a U_{wa}^2 A_L L_{OA} \tag{29}$$

where

$$U_{wa} = \sqrt{u_{wa}^2 + v_{wa}^2} \tag{30}$$

$$\psi_{wa} = \arctan \frac{v_{wa}}{u_{wa}} \tag{31}$$

$$u_{wa} = u + U_w \cos(\psi - \psi_w) \tag{32}$$

$$v_{wa} = v + U_w \sin(\psi - \psi_w) \tag{33}$$

and

$$\begin{aligned} C_X &= A_0 + A_1 \frac{2A_L}{L_{OA}^2} + A_2 \frac{2A_T}{B^2} \\ &\quad + A_3 \frac{L_{OA}}{B} + A_4 \frac{Sl}{L_{OA}} + A_5 \frac{C}{L_{OA}} \\ &\quad + A_6 M \end{aligned} \tag{34}$$

$$\begin{aligned} C_Y &= B_0 + B_1 \frac{2A_L}{L_{OA}^2} + B_2 \frac{2A_T}{B^2} \\ &\quad + B_3 \frac{L_{OA}}{B} + B_4 \frac{Sl}{L_{OA}} + B_5 \frac{C}{L_{OA}} \\ &\quad + B_6 \frac{A_{SS}}{A_L} \end{aligned} \tag{35}$$

$$\begin{aligned} C_N &= C_0 + C_1 \frac{2A_L}{L_{OA}^2} + C_2 \frac{2A_T}{B^2} \\ &\quad + C_3 \frac{L_{OA}}{B} + C_4 \frac{Sl}{L_{OA}} + C_5 \frac{C}{L_{OA}} \end{aligned} \tag{36}$$

MATHEMATICAL MODEL FOR LOW ADVANCE SPEED

Review of previous works and its applications

The MSS project summarized its contributions on its reports[4]. However, there are still many discussions especially on one of the conclusions concerning to the mathematical model at low advance speed. Some more detailed descriptions of proposed models are also found in [12], but here these models will be reviewed with some recent applications.

The major difference between the normal MMG-type mathematical model and the model at low advance speed is concentrated on the expression of hydrodynamic forces and moment acting on a hull;X_H, Y_H and N_H. They can be defined as follows in the case of eqs.(1-3).

$$\begin{aligned} X_H &= (m_y + X_{vr}) v_{ca} r + X_{vv} v^2 + X_{rr} r^2 \\ &\quad - R \end{aligned} \tag{37}$$

$$Y_H = Y_v v_{ca} + Y_r r + Y_{NL}(v_{ca}, r) \tag{38}$$

$$N_H = N_v v_{ca} + N_r r + N_{NL}(v_{ca}, r) \tag{39}$$

While MMG model treats nonlinear terms as 2nd or 3rd order polynomials of v and r for relatively small amplitude, the mathematical model at low advance speed should deal with wide range and variety of v and r. Besides, it should be noted u is as same amount as v and r, and further U becomes zero or minus. There are several approaches to solve these problems.

(1)Polynomial Models

If the continuity between MMG model is concerned, polynomial models are superior, though there are several ideas.

Hamamoto–Hasegawa model [13]

The continuity between MMG model and expansion for large amplitude of sway velocity are treated.

H. Kobayashi[14] for tug operation

E. Kobayashi–Asai model [15]

Four models are connected smoothly according to the advance speed.

E. Kobayashi[16] for various simulation at low advance speed

Takashina model [17]

The lateral force and moment with large sway velocity are expressed in Fourier expansion and converted into polynomial terms accounting major terms.

Hirano[18] for tug operation

Sohn[19] for various simulation at low advance speed

Kose model [20]

Standing the point that the ship speed U cannot be used for the non-dimensionalization of forces and moment, g is adopted instead. The continuity to MMG model should be studied.

Yamato, Koyama *et al.*[21] for berthing control

Fujino[22] for stopping manoeuvres

Shoji model [23]

Rewritten the forces and moment avoiding to divide by the ship speed U.

Shoji, Ohtsu and Mizoguchi[24] for berthing control

(2) Fourier Expansion Models

The Fourier expansion model is very natural from the experiment results of oblique towing tests. The forces and moment are mostly expressed in Fourier series. The problem is how to deal with yaw terms. It is also known this model is well expressed in polynomial model.

Takashina model [17]

Treating the yaw terms in 2nd order polynomials

Yumuro model [25]

Similar to Takashina model but employing only sine terms

Khattab model [26]

Treating rate of turn in effective sway velocity

(3) Cross-Flow Models

Cross-Flow model is based on the cross flow drag acting on a ship. However, the distribution of cross flow drag alongside a ship is hardly obtained or estimated, more precise estimation and at the same time simpler models are required.

Oltmann model [27]

Dividing the hydrodynamic forces and moment into some physical meaningful terms

Karasuno model [28]

Similar to Oltmann model, but simplified it

Yoshimura model [29]

Treating C_D as a constant alongside a ship

(4) Response Equation Model

Nomoto's K–T model is still widely used to predict ship manoeuvrability, especially at full-scale trials. The model is expanded to 2nd order K–T model and its nonlinear model for VLCC/ULCC's. This model is also expandable at low advance speed.

Yoshimura–Nomoto model [30]

This model is maybe the first model treating various advance speed with various propeller revolution. The model is very simple and easily implemented into an analog computer, so it is used for first generation ship handling simulators.

Aiguo[31] for ship handling simulator

In this study, we adopted several polynomial models for simulation program, but here Kose model will be briefly introduced in the next subsection.

Hydrodynamic forces and moment acting on a hull at low advance speed

Non-dimension convention

Kose*et al.*[20] proposed the non-dimension factor for force as $\rho/2 \cdot L^3 g$ instead of $\rho/2 \cdot L^2 U$. This is because to avoid the divergence on terms divided by U, when the ship advance speed is zero. The other physical terms are non-dimensionalized in the following ways.

$$X^*, Y^* = X, Y/\frac{\rho}{2}L^3 g \quad (40)$$
$$N^* = N/\frac{\rho}{2}L^4 g \quad (41)$$
$$m^* = m/\frac{\rho}{2}L^3 \quad (42)$$
$$I^*_{zz} = I_{zz}/\frac{\rho}{2}L^5 \quad (43)$$
$$u^*, v^* = u, v/\sqrt{Lg} \quad (44)$$
$$r^* = r/\sqrt{L/g} \quad (45)$$
$$\dot{u}^*, \dot{v}^* = \dot{u}, \dot{v}/g \quad (46)$$
$$\dot{r}^* = \dot{r} \cdot L/g \quad (47)$$

Table 1 . Principal particulars.

Item	*Symbol*	*Value*	*Unit*
Ship length	L	304.0	m
Length overall	L_{OA}	310.5	m
Ship breadth	B	52.5	m
Mean draught	d_m	17.4	m
Block coefficient	C_b	0.827	–
Mass	m	2.350 $\times 10^8$	kg
Moment of inertia	I_{zz}	1.018 $\times 10^{12}$	kgm^2
Wetted surface area	S	2.259 $\times 10^4$	m^2
Propeller diameter	D_P	8.5	m
Propeller pitch	P	5.16	m
Rudder area	A_R	98.0	m^2
Rudder height	h	12.94	m
Rudder aspect ratio	Λ_R	1.709	–

Expression of hull forces and moment

Transverse and lateral forces and moment around z-axis are modelled in the following equations.

$$\begin{aligned} X^*_H &= X^*_{vr}v^*r^* + X^*_{uu}|u^*|u^* \\ &\quad + X^*_{uvv}u^*v^{*2}/U^* + X^*_{vvr}|v^*|v^*r/U^* \quad (48) \\ Y^*_H &= Y^*_v U^* v^* + Y^*_{vv}|v^*|v^* + Y^*_{vvvvv}v^{*5}U^{*3} \\ &\quad + Y^*_r r^* + Y^*_{ur}u^*r^* + Y^*_{uvvr}u^*v^{*2}r^*/U^{*2} \\ &\quad + Y^*_{vrr}v^*r^{*2}/U^* \quad (49) \\ N^*_H &= N^*_{uv}u^*v^* \\ &\quad + N^*_r r^* + N^*_{rrr}r^{*3} \\ &\quad + N^*_{ur}u^*r^* + N^*_{vvr}v^{*2}r^* \quad (50) \end{aligned}$$

In the eqs.(48–50), u and v should be replaced with u_{ca} and v_{ca} defined in eqs.(4–5), where current exists.

BERTHING CONTROL USING NEURAL NETWORK

Berthing is one of the most difficult manoeuvres in ship operation. A captain or a pilot should takes care of the ship's manoeuvrability, current and wind conditions, harbour configuration, other traffic and tug operations. If all or some part of berthing is automated or guided by a computer, it will be of great help for them.

In this section, an approach for it is shown as an application of mathematical model at low advance speed.

Ship particulars and derivatives

A tanker is considered for the simulation. In Table 1, the principal particulars of the tanker is shown. The constants concerning to the wind resistance are listed on Table 2. Hydrodynamic derivatives concerning to X_H, Y_H and N_H are listed on Table 3.

Neural control of berthing

Neural controller was designed for berthing control. Three layer network with error back propagation

Table 2 Wind resistance constants.

Symbol	*Value*	*Unit*
A_L	4135.6	m^2
A_T	993.5	m^2
A_{SS}	1047.5	m^2
C	164.5	m
Sl	416.7	m
M	2	–

Table 3 Hydrodynamic derivatives.

X^*_{vr}	-0.310×10^{-2}
X^*_{uu}	-0.457×10^{-3}
X^*_{uvv}	2.927×10^{-3}
X^*_{vvr}	-0.558×10^{-2}
Y^*_{v}	-2.222×10^{-2}
Y^*_{vv}	-2.173×10^{-2}
Y^*_{vvvvv}	0.0
Y^*_{r}	1.287×10^{-5}
Y^*_{ur}	0.510×10^{-2}
Y^*_{uvvr}	-0.851×10^{-2}
Y^*_{vrr}	-0.874×10^{-2}
N^*_{uv}	-7.910×10^{-3}
N^*_{r}	-1.632×10^{-4}
N^*_{rrr}	-0.776×10^{-2}
N^*_{ur}	-1.584×10^{-3}
N^*_{vvr}	-0.981×10^{-1}

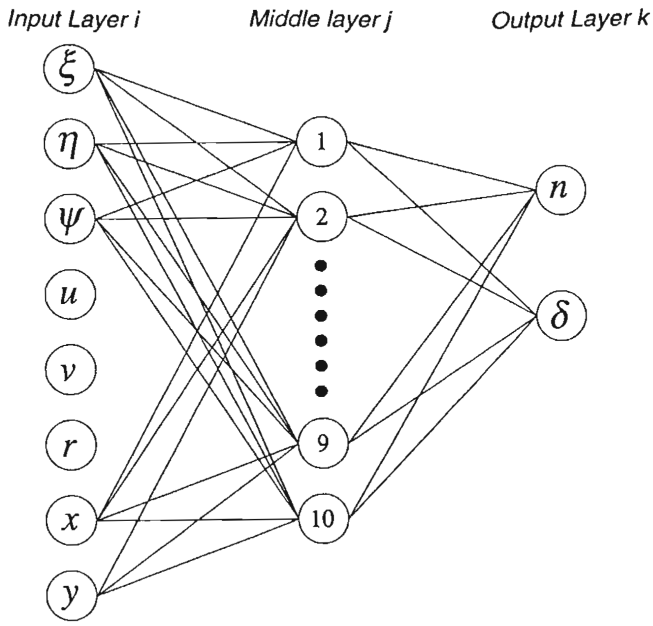

Figure 2 Three layer neural network.

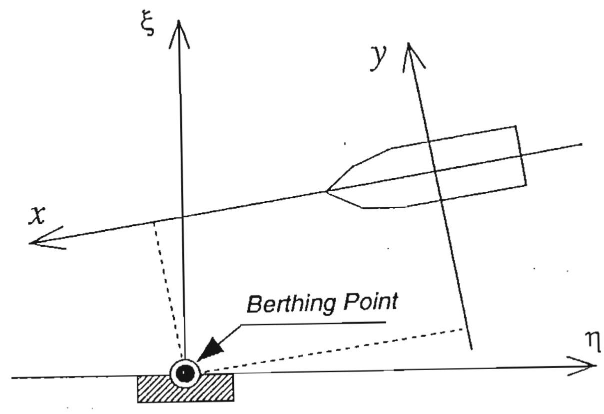

Figure 3 Definition of ξ and η.

method is employed. Similar network was used by Yamato *et al.*[32], but some modification to improve the performance was done. The detail description will be found in [33]. Neural network is designed as shown in Figure 2, where left-hand side neurons are inputs layer, middle column neurons are middle layer and right-hand side neurons are outputs layer. As inputs, 7 variables as shown in the figure are used, where ξ and *eta* are defined as shown in Figure 3.

Learning parameters

After providing teaching data, which was taken from the simulation results of manual berthing done by one of the authors, network parameters were determined by learning. Learning procedure was finished by monitoring the root mean square value of errors. Figure 4 shows 7 patterns used for teaching data. Figure 5 is one of the example of teaching data (○ denotes a sampled point used for learning).

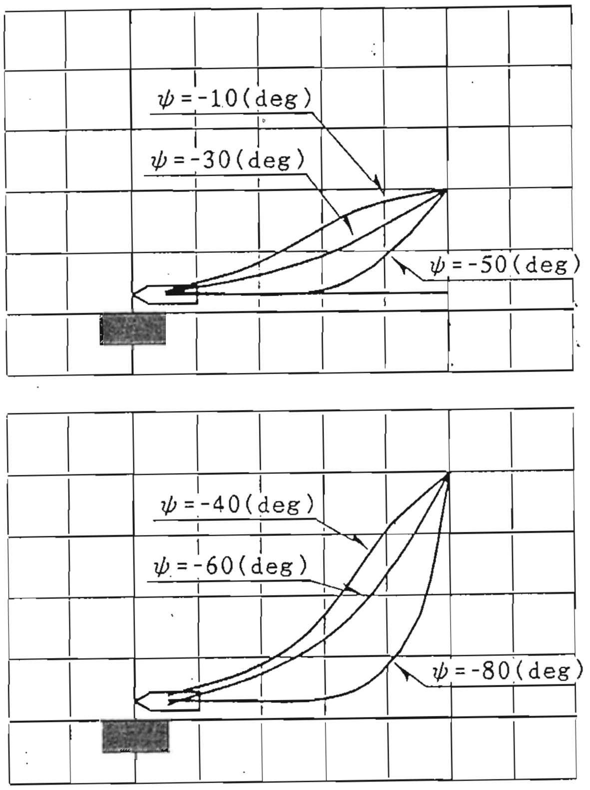

Figure 4 Patterns of teaching data.

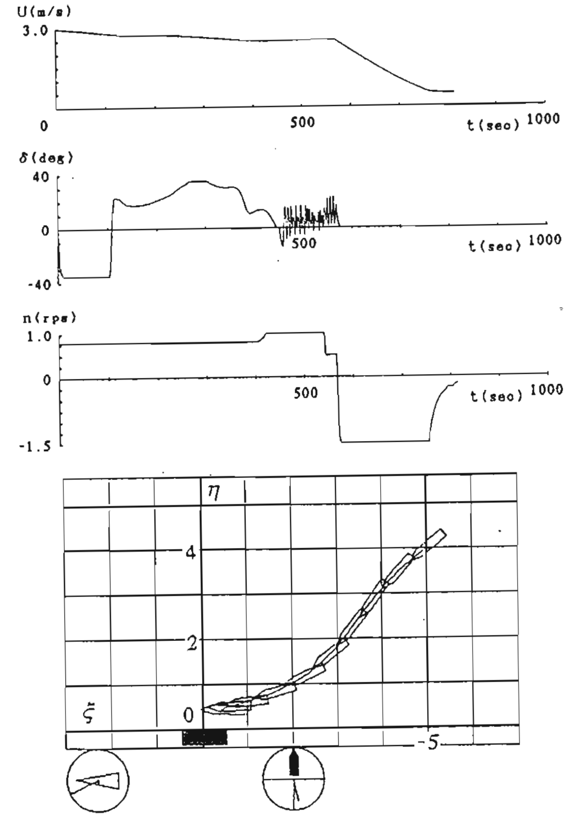

Figure 6 Simulation starting at (5L, 4L) with ψ=-40deg.

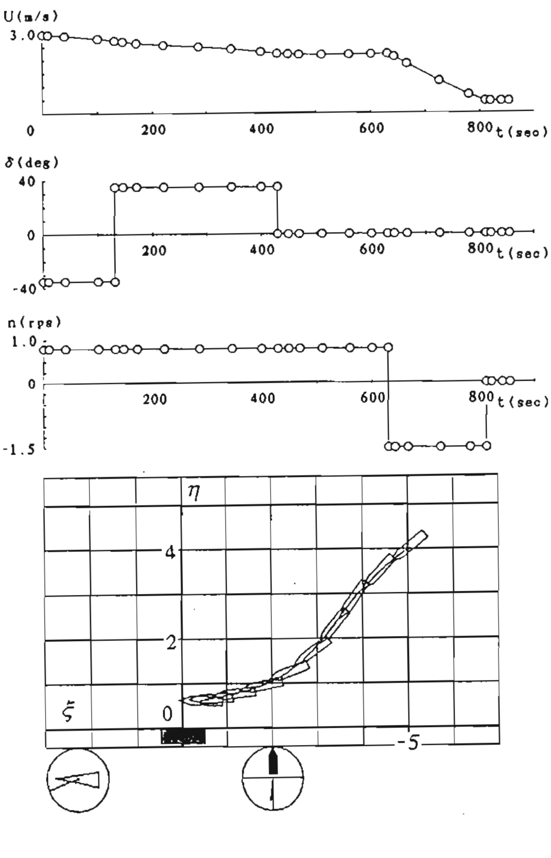

Figure 5 An example of given teaching data: starting at (5L, 4L) with ψ=-40deg.

Results of simulation

Simulation of automatic berthing is done with various initial conditions including those of teaching data. In this paper, berthing manoeuvres phase I (defined by Yamato [32]) is treated; *i.e.* when the ship approaches to the pear inside a given area (here $L/2$ square) with less than a given velocity (here 0.5 m/sec), the control task is regarded as being finished. At the initial conditions as same as those of teaching data, the results are of course satisfactory. Figure 6 shows a result of the same condition as Figure 5. The trajectory looks almost same as that of teaching data, but in the time history we can notice that the neural controller adjusted the control (of rudder and propeller revolution) so as to follow the teaching trajectory. The neural controller has an advantage when the initial conditions (entering point, angle and velocity) have changed. It is usually difficult to adjust parameters for such conditions in the cases of ordinary control methods. Figure 7 (entering angle is different), Figure 8 (entering position is different) and Figure 9 (entering position is different) verify this. Wind effect is also checked. Figure 10 (heading wind), Figure 11 (lee wind) and Figure 12 (following wind) show the effect of various wind direction. (The needle in the above circle shows the wind direction.) The controller adjusts the amount or the timing of commands according to the wind condition.

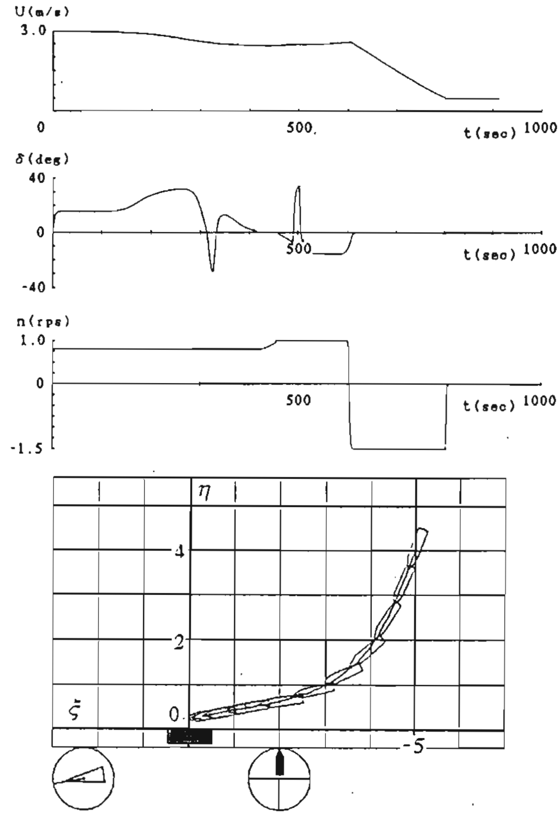

Figure 7 Simulation starting at (5L, 4L) with ψ=-70deg.

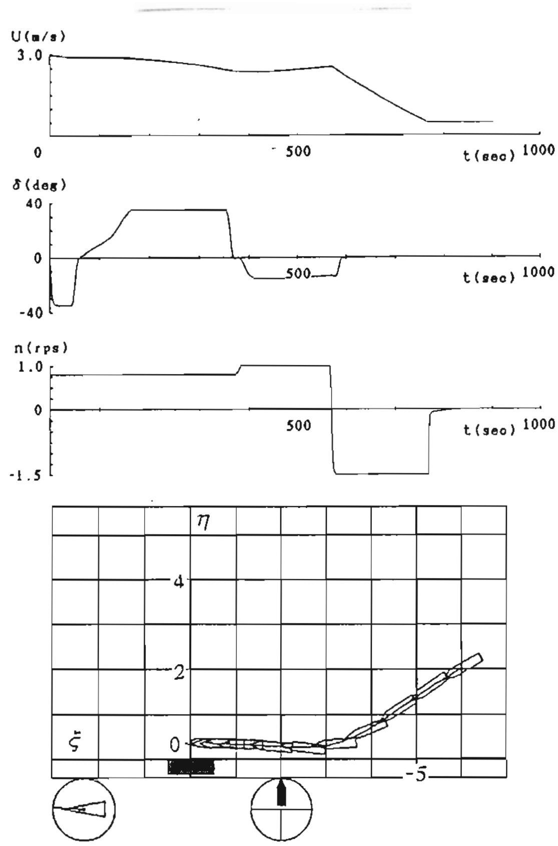

Figure 9 Simulation starting at (6L, 2L) with ψ=-30deg.

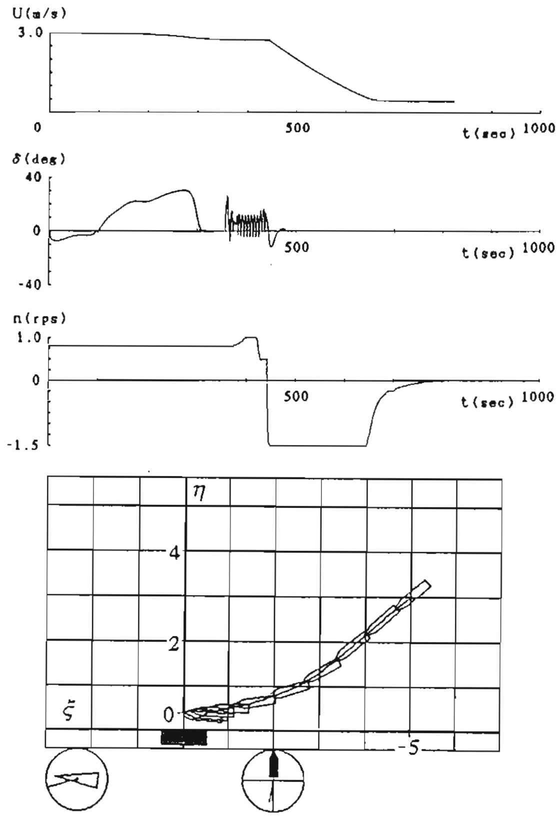

Figure 8 Simulation starting at (5L, 3L) with ψ=-40deg.

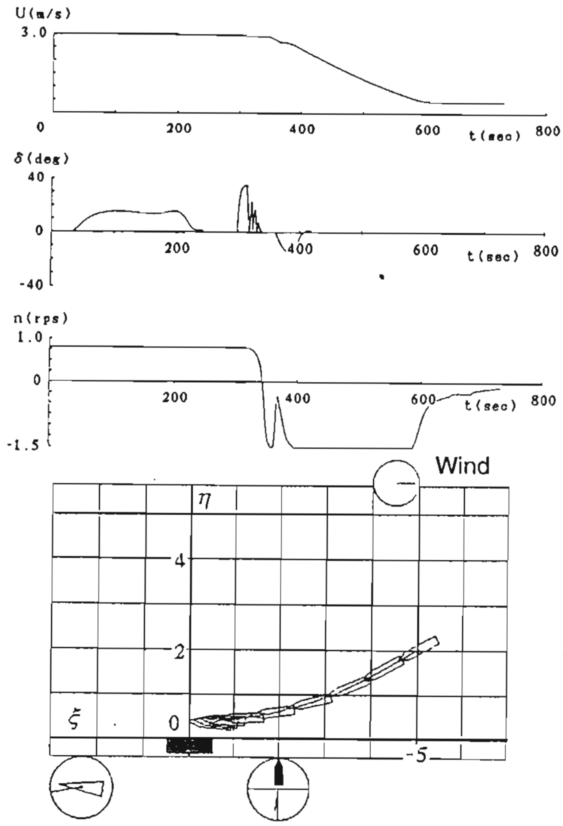

Figure 10 Simulation starting at (5L, 3L) with ψ=-30deg (fair wind).

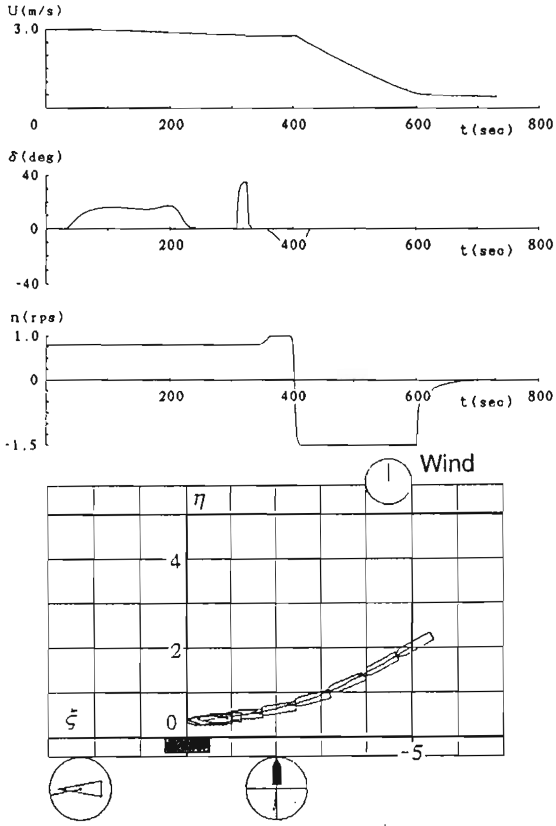

Figure 11 Simulation starting at ($5L$, $3L$) with ψ=−30deg (side wind).

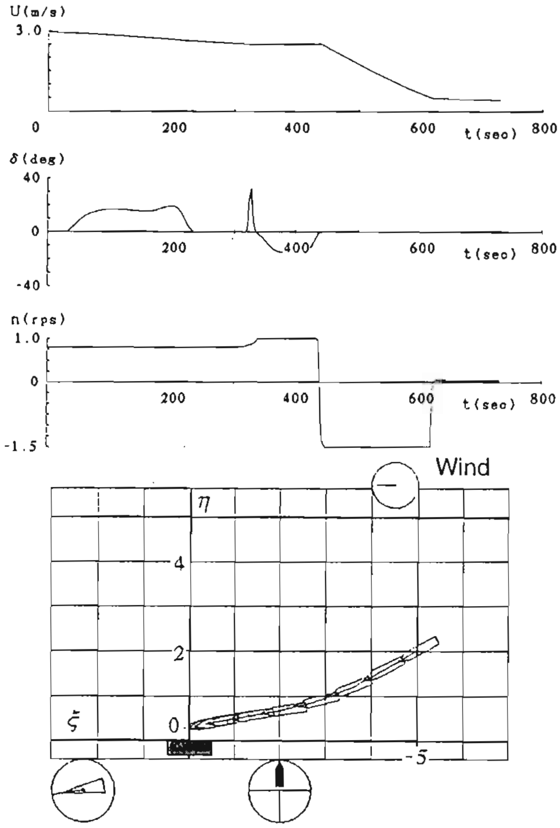

Figure 12 Simulation starting at ($5L$, $3L$) with ψ=−30deg (contrary wind).

CONCLUSIONS

Mathematical models of ship manoeuvring at low advance speed are reviewed mainly from Japanese papers. Kose model is chosen from them and applied for berthing control.

The main conclusions obtained are summarized as follows:

1. Mathematical models of ship manoeuvrability at low advance speed are summarized with recent applications.
2. Neural controller designed for berthing control is verified to work satisfactory for various conditions.

The following points are the recommendation for the future study.

1. More researches on mathematical model at low advance speed are required to standardize it.
2. More captive and free-running models tests should be done for various types of ships to gather derivatives for the model.
3. It is required to establish the method to estimate the derivatives of the model from the practical purpose.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to Prof. K. Kose, Hiroshima University and Prof. H. Hinata, Maritime Safety Academy for providing the data of their experiments.

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NOMENCLATURE

Physical constants

g : gravity acceleration
ρ_a : density of air
ρ : density of water

Principal particulars

L : ship length between parpendiculars
L_{OA} : ship overall length
B : ship breadth
d_m : ship mean draught
τ : trim
Λ : ship aspect ratio

Mass and added masses

m : mass of a ship
m_x : longitudinal added mass of a ship
m_y : transverse added mass of a ship
I_{zz} : moment of inertia around z–axis of a ship
J_{zz} : added moment of inertia around z–axis of a ship

Velocities and angles

u : longitudinal component of ship velocity U
v : transverse component of ship velocity U
r : rate of turn around z-axis of a ship
ψ : ship direction
$\delta*$: rudder command
δ : rudder angle

Current parameters

U_c : current velocity
ψ_c : current direction
u_{ca}, v_{ca} : longitudinal and transverse relative flow velocities by current

Wind parameters

U_w : wind velocity
ψ_w : wind direction
u_{wa}, v_{wa} : longitudinal and transverse relative velocities by wind
U_{wa} : relative velocity by wind
ψ_{wa} : relative direction of wind

Thrust and resistance

T : propeller thrust
R : longitudinal resistance of hull with rudder
t : thrust deduction coefficient
t_R : deduction coefficient of rudder nominal force

Hydrodynamic forces

Y_{NL}, N_{NL} : nonlinear hydrodynamic force (transverse) and moment (around z–axis)
C_D: cross-flow drag coefficient

Rudder force

F_N : nominal force acting on a rudder

Wind resistance coefficients

X_w, Y_w, N_w : aerodynamic forces (longitudinal and transverse), and moment (around z–axis) acting on a ship by wind
C_X, C_Y, C_N : coefficients (longitudinal, transverse forces and moment) of wind resistances
A_L : lateral projected area of a ship above waterline
A_T : transverse projected area of a ship above waterline
A_{SS} : lateral projected area of superstructure
Sl : length of perimeter of lateral projection of a ship excluding waterline and slender bodies such as masts and ventilators
C : distance from bow of centroid of lateral projected area
M : number of distinct groups of masts or kingposts seen in lateral prjection; kingposts close against the bridge front are not included
$A_0 \cdots A_6$, $B_0 \cdots B_6$, $C_0 \cdots C_6$: regression coefficients of wind resistances[11]

Steering gear parameters

T_E : time constant of a steering gear (sec)
$\dot{\delta}_{\max}$: maximum rudder speed (deg/sec)