A Consideration on the Governing Equation of the Ship's Maneuvering Motion in Waves

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ABSTRACT

The traditional governing equations for the sway-yaw maneuvering motion are a set of ordinary differential equations with constant coefficients. But, as well known, the integro-differential equations with impulse response functions are more strict governing equations that can handle the frequency dependence of hydrodynamic forces.

In this paper, the two types of equations are compared and used to calculate the 10° - 10° *zig · zag* maneuver in waves, and the differences of the solutions are discussed.

INTRODUCTION

The maneuvering motion of a ship in waves, contains some difficulties from the hydrodynamic point of view. The changes of yaw angle and forward speed result in the change of encounter frequency of waves. As a result, the hydrodynamic forces exerted on ship are varied continously.

Meanwhile, the response of a ship to the rudder deflection is very slow and it becomes difficult to determine the suitable frequency of hydrodynamic forces when we treat the problems with the traditional ordinary differential equations.

A lot of researchers have pointed out that it is not correct to treat such problems with differential equations of constant coefficients. They also have referred the integro-differential equations using impulse response functions of hydrodynamic forces. (1 - 5)

Fujino has made sway-yaw integro-differential equation with impulse responses gotten through the Fourier transformation of PMM data of addeded masses and damping coefficients in wide range of frequency. And he compared the differences between the ordinary differential treatment and the integro-differential calculation, when the stepwise rudder is applied in both cases. He showed the responses of the ship are almost same in both approaches.(6)

Perez also has made the sway-yaw-roll equation with linear impulse response functions of wave exciting forces. He used the strip method to calculate the wave forces in frequency domain, and later these are used to calculate the impulse response functions.(7)

In this paper, the linear integro-differential sway-yaw equations with rudder deflection in regular waves are derived, and the maneuvering motion of Todd's series 60 model is calculated. The differences between the integro-differential equation and ordinary differential equation are discussed by comparing the response of typical maneuver like zig-zag test.

Here, the indirect method using Fourier transformation, is adopted to get the impulse responses of radiation forces and wave exciting forces.

EQUATION OF SWAY-YAW LINEAR MOTION

Using the coordinate system of Fig.1, the sway-yaw equations with constant coefficients are written as follows.

$$(m-Y_{\dot v})\dot v - Y_v v + (mU - Y_r)r - Y_{\dot r}\dot r = Y_\delta \delta + Y_{wc}\cos w_e t + Y_{ws}\sin w_e t$$
$$(I_{zz}-N_{\dot r})\dot r - N_r r - N_v v - N_{\dot v}\dot v = N_\delta \delta + N_{wc}\cos w_e t + N_{ws}\sin w_e t \qquad (1)$$

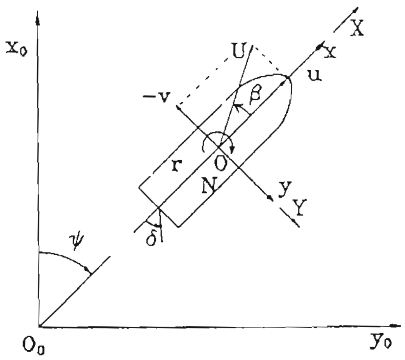

Fig.1 Coordinate System

Meanwhile,the strict sway-yaw equation can be built as the Volterra Functional forms. Here, the kernels are the impulse responses of the hydrodynamic forces.

$$m(\dot v + Ur) + \int_{-\infty}^{t} K_v(\tau)v(t-\tau)d\tau + \int_{-\infty}^{t} K_r(\tau)r(t-\tau)d\tau = + \int_{-\infty}^{t} K_\delta(\tau)\delta(t-\tau)d\tau + \int_{-\infty}^{t} K_\zeta(\tau)\zeta(t-\tau)d\tau$$
$$I_{zz}\dot r + \int_{-\infty}^{t} M_v(\tau)v(t-\tau)d\tau + \int_{-\infty}^{t} M_r(\tau)r(t-\tau)d\tau = \int_{-\infty}^{t} M_\delta(\tau)\delta(t-\tau)d\tau + \int_{-\infty}^{t} M_\zeta(\tau)\zeta(t-\tau)d\tau \qquad (2)$$

Impulse response functions on the left side of eq.(2), having singularities at τ=0, transformed into eq.(3) which has only regular functions.

$$(m - Y_{\dot v}(\infty))\dot v - Y_v(\infty)v - Y_{\dot r}(\infty)\dot r + (mU - Y_r(\infty))r + \int_0^\infty K_v^*(\tau)v(t-\tau)d\tau + \int_0^\infty K_r^*(\tau)r(t-\tau)d\tau = + Y_\delta\delta + \int_0^\infty F_\zeta(\tau)\zeta(t-\tau)d\tau \qquad (3)$$

$$(I_{zz}-N_{\dot r}(\infty))\dot r - N_r(\infty)r - N_{\dot v}(\infty)\dot v + N_v(\infty)v + \int_0^\infty M_v^*(\tau)v(t-\tau)d\tau + \int_0^\infty M_r^*(\tau)r(t-\tau)d\tau = + N_\delta\delta + \int_0^\infty F_\zeta(\tau)\zeta(t-\tau)d\tau$$

In eq.(3), the impulse response functions due to rudder deflection are treated as constant, because no avaliable data that can calculate the rudder impulse responses exist.

$$\int_{-\infty}^{t} K_\delta(\tau)\delta(t-\tau)d\tau = \delta\int_{-\infty}^{\infty} K_\delta(\tau)d\tau = \delta\cdot Y_\delta \qquad (4)$$

$$\int_{-\infty}^{t} M_\delta(\tau)\delta(t-\tau)d\tau = \delta\int_{-\infty}^{\infty} M_\delta(\tau)d\tau = \delta\cdot N_\delta$$

CALCULATION OF IMPULSE RESPONSE FUNCTIONS

1 Impulse response functions of radiation forces

The dircet method to get the impulse responses of moving ship, is to solve the boundary value problem in time domain. But the procedure is very complicated and difficult. Another way to get the impulse response functions is using the Fourier transform of the frequency responses of radiation forces. The latter method is indirect but simple.

In this paper, the P.M.M. data of Todd's series 60 model by Van Leeuwen[8] are used to calculate the impulse responses due to sway velocity and yaw angular velocity.

That is,

$$K_v^*(\tau) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\{Y_v(\omega)-Y_v(\infty)\}\cos\omega\tau d\omega \qquad (5)$$

$$= -\frac{1}{\pi}\int_{-\infty}^{\infty}\{(m-Y_{\dot v}(\omega))-(m-Y_{\dot v}(\infty))\}\sin\omega\tau d\omega$$

$K_r^*(\tau)$, $M_v^*(\tau)$, $M_r^*(\tau)$ can be calculated in the same way.

Fig.2 shows the impulse response functions of the given model at Froude number 0.2. Here, the added masses and damping coefficients are fitted with rational functions to get the exact Fourier transformation. And, the impulse response functions gotten from Fourier cosine transformation are exactly coincided with the results of Fourier sine transformation.

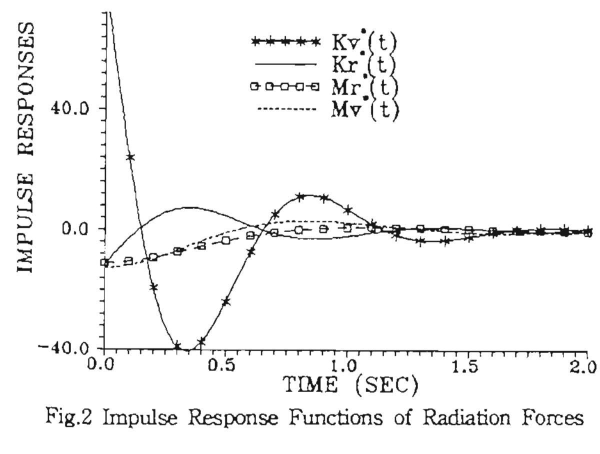

Fig.2 Impulse Response Functions of Radiation Forces

2 Impulse response functions due to waves

The impulse response functions by impulsive wave with unit amplitude, can be calculated through the Fourier transformation of the responses of model to the various sinusoidal waves in frequency domain. In this paper, O.S.M.(Ordinary Strip Method) is used to calculate the frequency responses of wave forces of a given model.

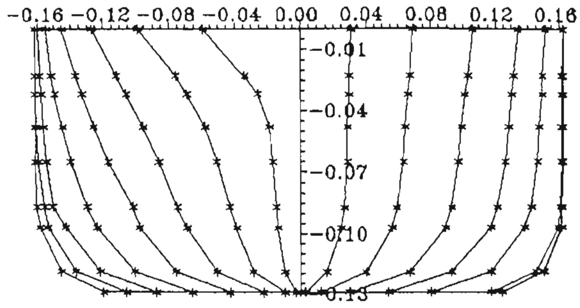

Fig.3 Offsets of Series 60 (C_b = 0.7)

For the twenty-one 2-dimensional sections in Fig.3, the added mass and wave making damping coefficent are calculated by the Close-fit method[9].

Fig.4 shows the response characteristics of sway force and yaw moment, by the regular waves coming with encounter angle $\mu=150°$ and amplitude $\zeta_a=1.0m$.

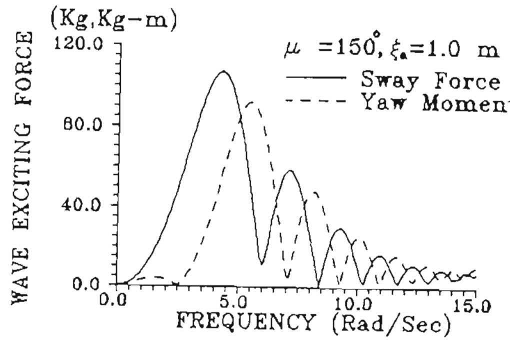

Fig.4 Frequency Responses of Wave Exciting Forces

Here, the impulse response functions by waves are

$$K_\zeta(t) = \frac{2}{\pi}Re\{\int_0^\infty H_f(\omega)\cdot e^{i\omega t}d\omega\}$$

$$M_\zeta(t) = \frac{2}{\pi}Re\{\int_0^\infty H_M(\omega)\cdot e^{i\omega t}d\omega\} \qquad (6)$$

In eq.(5), $H_f(\omega)$ and $H_M(\omega)$ means the frequency response of sway force and yaw moment , respectively. The impulse response functions gotten in such way, presented in Fig.5.

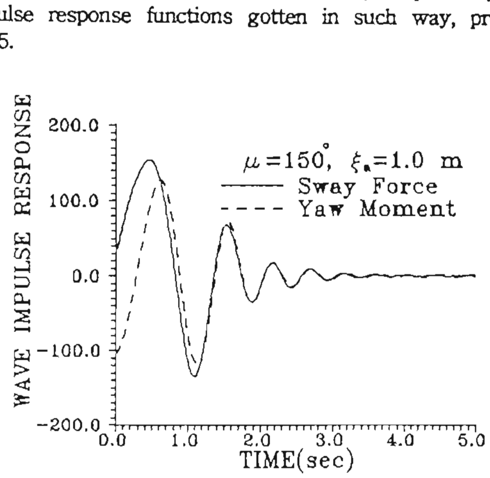

Fig.5 Impulse Response Functions of Wave Exciting Forces

In usual maneuvering motion, the heading angle changed successively and the encounter angle of incident waves also varies continously. The hydrodynamic forces of the ship by waves are functions of encounter frequency ω_e, but in this paper the authors neglect this fact and the encounter angle is fixed at $\mu = 150°$ for the following calculations, to simplify the problems.

EXAMINATION ON THE CALCULATION RESULTS

In this paper, subjects are restricted to the linear sway-yaw motion , so the authors confined the simulations to the 10° - 10° *zig · zag* maneuvers.

At first, the usual calculation by eq.(1) and the elaborate calculation through the eq.(2) in calm sea, is simulated and compared in Fig.6 .

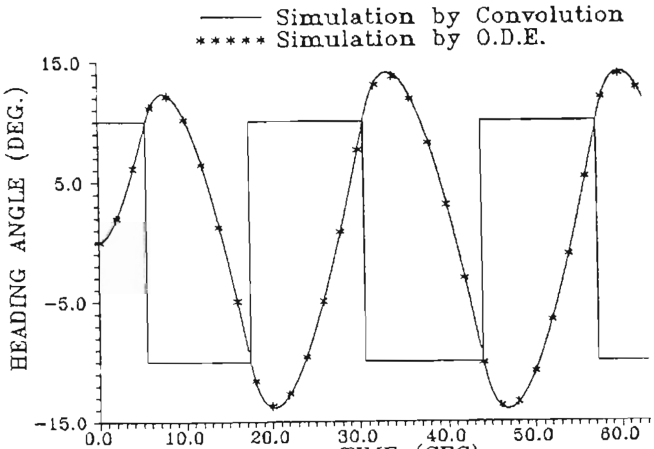

Fig.6 10° - 10° *zig · zag* with Rudder Deflection only

The hydrodynamic derivatives and the principal dimensions for the model, are listed on the Table1 and Table 2, respectively.

Table.1 Linear Maneuvering Derivatives

at $\omega \fallingdotseq 0$, $\omega = \infty$ (Fn = 0.2)

Derivative	$\omega \fallingdotseq 0$	$\omega = \infty$
$m' - Y\dot{v}'$	0.0229	0.01542
Yv'	- 0.0222	- 0.052552
$Y\dot{r}'$	- 0.00039	0.000186
$m' - Yr'$	0.0076	0.011
$Izz' - N\dot{r}'$	0.0012	0.00092
Nr'	- 0.0034	- 0.00933
$N\dot{v}'$	- 0.00048	- 0.000073
Nv'	- 0.0057	0.0037
Y_δ'	- 0.00211	
N_δ'	0.001	

Table.2 Principle Dimensions of Model

Lpp	2.258m
B	0.323m
d	0.129m
W	65.714kgf
k_{zz}'	0.25

Runge-Kutta-Gill's method is used to simulate the *zig · zag* motion. The time step is dt = 0.01 *sec*, and the rudder is deflected in stepwise manner. As in the Fig.6 , no visible differences can be found between two calculations.

Next, for the simulation of the *zig · zag* maneuver in regular waves, the wave exciting forces are calculated by convolution integral and presented on the Fig.7 ~ Fig.8. Here, the convolution integral was carried out for 5 seconds and time interval 0.01 sec, in Simpson's rule.

In the figures, dotted lines mean the fitted values of wave forces to the sinusoidal functions. Later, these are used as the regular exciting forces in the ordinary differential equation.

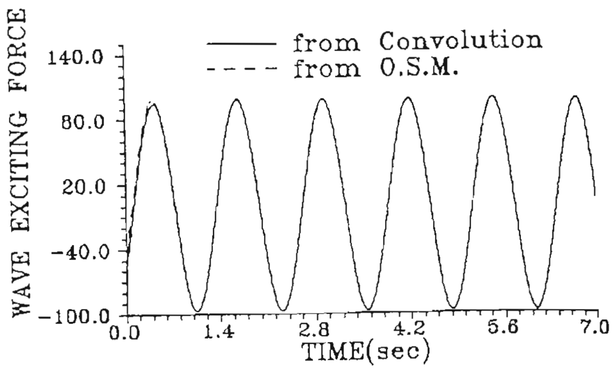

Fig.7 Wave Exciting Force used in the Simulation

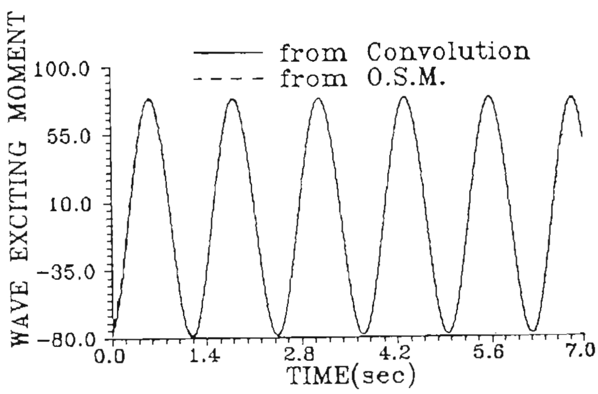

Fig.8 Wave Exciting Moment used in the Simulation

By doing this, the exciting forces in the ordinary differential equation and in the integro-differential equation become equal.

The 10° - 10° *zig · zag* simulation results by eq.(1) and by eq.(2), in regular waves are shown in Fig.9 - Fig.10. In the figures, we can see that the difference between two calculations are very small.

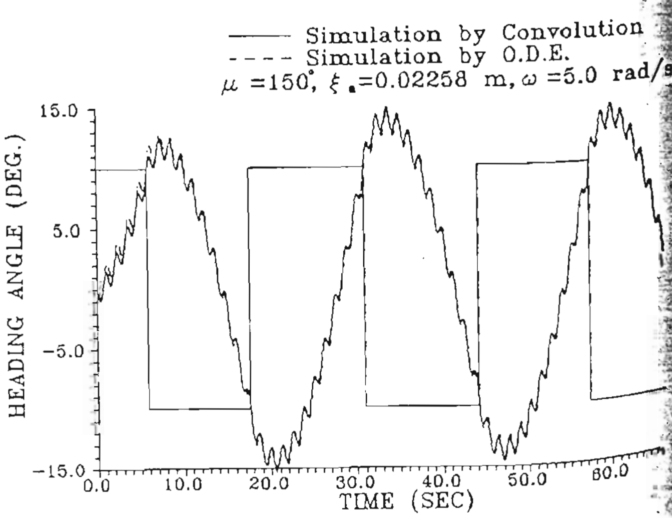

Fig.9 10° - 10° *zig · zag* in Regular Waves

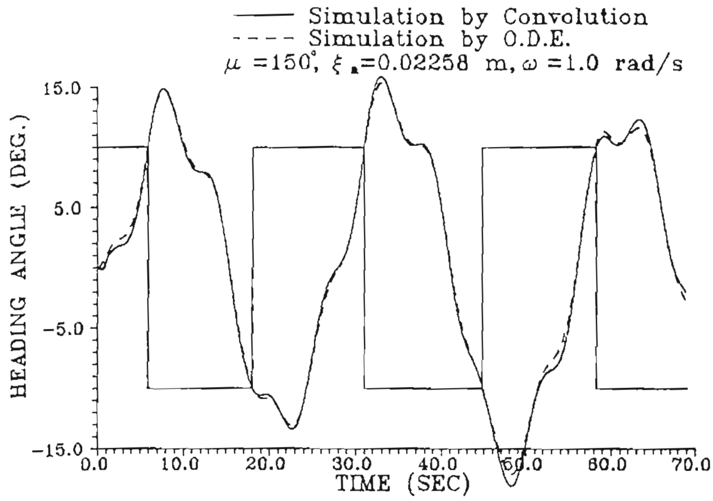

Fig.10 10° - 10° *zig · zag* in Regular Waves

CONCLUSIONS

For the 2.258 m model of Todd's Series 60 (C_b=0.7), two governing equations of maneuvering motion in waves, that is, the ordinary differential equation and the integro-differential equation, are examined from the point of linear physical system. The results of examination are as follows.

(1) Confined to the linear maneuvering motion in waves, the usual constant coefficients equation gives good agreement with the exact integro-differential equation.

(2) The calculation in this paper was for a model which has good directional stability and ordinary hull form. Therefore, the same calculations must be carried out for the blunt or directionally unstable ship ,to make the conclusion (1) more general.

(3) The extension of present topic to the nonlinear maneuvering motion, such as turning in waves, needs more strict considerations.

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