31

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Learning Aspects of a Fuzzy Controller for an Inverted Pendulum

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Abstract: The controller of an inverted pendulum is designed, focussed on the realization aspects of a fuzzy controller of considerable complexity and the importance of learning in such a situation. The features of this problem are: its inherent instability, its nonlinearity and the fact that two outputs should be controlled by one input signal. The paper describes experiments in which the controller has been tested. These were carried out in a real-time simulation and on the experimental setup. For real-time capability, we introduced transputers (parallel processors) hosted by an IBM-PC. The paper ends with a discussion on the possibility of using learning algorithms to fill in the fuzzy rule base in areas where rules were not found by the designer and to improve the rules found by the designer.

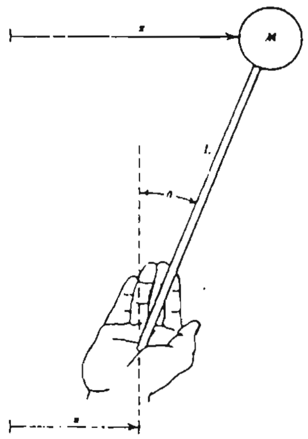


Fig. 1 Manual balancing of a stick

1. INTRODUCTION

The controller design is one of the oldest, and at the same time, one of the newest topics in mechanics and mechanism. The history of the control theory verifies this. James Watt opened a way of feedback control using a mechanism itself in 1788. Development of modern control theories have pursued the optimization of a given cost function and recent applications of fuzzy logics and/or neural networks are reflecting the fact that we should and still can learn more from the human behaviours.

In this paper we are dealing with a classic control problem; an inverted pendulum. The task of this problem is to keep a pole or a stick in upright condition by moving the end of the stick. Usually we, human beings, put the stick on our palms and shift palms mainly in the horizontal plane as shown in Fig. 1. However, when we make the experimental setup of this problem, several systems are available and proposed already. The widely used model is so-called cart-pole system as shown in Fig. 2. It consists of a cart that can move along a track. On the cart a stick is mounted that can rotate freely. By applying a force to the cart and thereby moving it, the stick can be balanced.

Many researches were done to investigate or validate several control methods. Michie and Chambers [1] designed a learning controller for this problem, using this setup. Barto *et al.*[2] used this setup to implement a neural network. The neural network works well in simulation. Recently, Kumagai *et al.* [3] have shown the result of the neural networks in the real system for this setup. How-

*This research was done at University of Twente during his stay of Aug. 1991 – Apr. 1992.

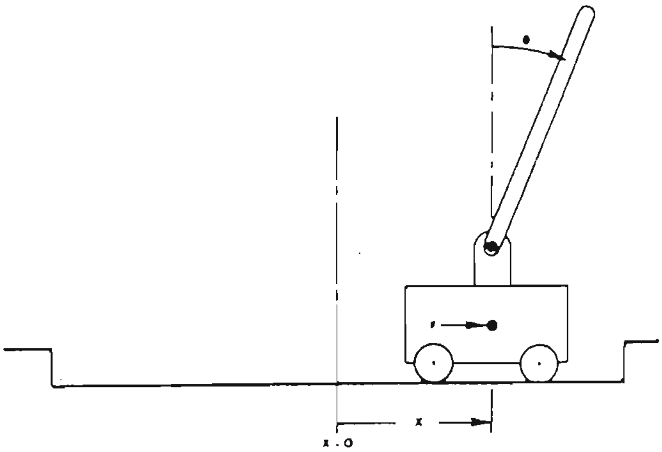


Fig. 2 Cart-pole system

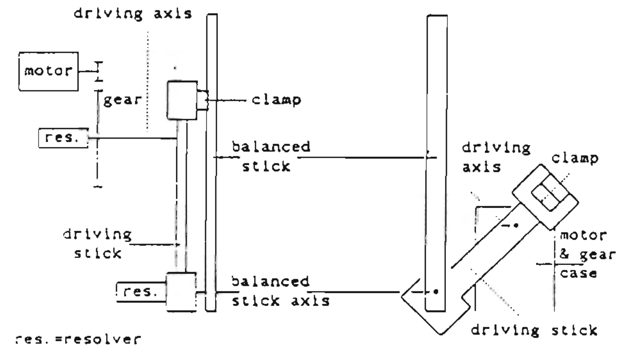


Fig. 3 The balanced stick system

ever, problems still remain on the following summarized points:

- The controller behaviour in the real system
- The learning rate
- The robustness and the effect of nonlinearity

We have made another setup for this problem as shown in Fig. 3, which involves more nonlinear effect in the real and simulation systems. Several control methods, including statefeedback control, manual control and neural control were tested for this setup, before considering its learning capability. Fuzzy controller was taken for the start-up controller and manual control experiments were executed to extract human control model. Finally, discussions are done for the future learning controller.

2. DESCRIPTION OF THE PROBLEM

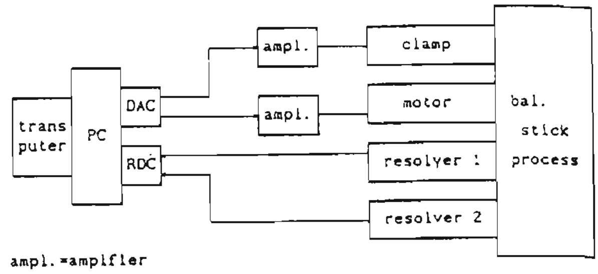


Fig. 4 Block diagram of the balanced stick system

2.1 Hardware of the System

As shown in Fig. 3, it consists of two sticks. One is the driving stick and is connected on a shaft on the driving axis. This shaft is driven by an electrical motor. The balanced stick is connected at one end of the driving stick, which we call the balanced stick axis, in parallel with the driving axis. Both sticks can rotate only in the plane perpendicular to the axes. The balanced stick has to be balanced by applying a torque on the motor axis. In practice, the motor is not connected directly to the driving axis. To increase the torque on the driving axis, the motor is coupled to the driving axis by a gear with gear ratio of 10:1.

The motor is controlled by a computer, an IBM-PC compatible. The computer is equipped with a 16-bit DAC (Digital to Analog Converter). The output of the DAC is amplified and converted to a current and then feeds to the motor.

On the driving axis and on the balanced stick axis resolvers are mounted to measure the angles. One resolver mounted on the driving axis measures the angle of the driving stick, which we call ϕ, and the other mounted on the balanced stick axis measures the angle between the driving stick and the balanced stick. As we define the angle between the balanced stick and a fixed vertical line as θ, the second resolver measures the angle $\phi - \theta$.

To simplify the positioning the balanced stick upright, a clamp is mounted on the driving stick. If the balanced stick is in line with the driving stick, the clamp can keep it in this position. If the clamp is pulled back, the balanced stick can rotate freely. The clamp is also controlled by the computer.

For fast computation, the computer is also equipped with a transputer board, on which 7 INMOS T800 transputers can be used. The complete system is shown in Fig. 4.

2.2 Modelling of the System

To make the system equations, bond graph is used as shown in APPENDIX. Here the results are shown.

$$\ddot{\phi} = \Big\{\Big(-m_s l_b l_s \cos(\phi-\theta)\Big)\Big[\Big(R_s + m_s l_b l_s \sin(\phi-\theta)\dot{\phi}\Big)\dot{\phi} - R_s\dot{\theta} - l_s m_s g\cos\theta\Big]$$
$$+\Big(J_s + m_s l_s^2\Big)\Big[-\Big(R_d' + R_s\Big)\dot{\phi} + \Big(R_s - m_s l_b l_s \sin(\phi-\theta)\dot{\theta}\Big)\dot{\theta} + M_d$$
$$-\Big(l_d m_d + l_b m_s\Big)g\cos\phi\Big]\Big\}/\,\mathrm{Det} \quad (1)$$

$$\ddot{\theta} = \Big\{\Big(-m_s l_b l_s \cos(\phi-\theta)\Big)\Big[-\Big(R_d' + R_s\Big)\dot{\phi} + \Big(R_s - m_s l_b l_s \sin(\phi-\theta)\dot{\theta}\Big)\dot{\theta} + M_d$$
$$-\Big(l_d m_d + l_b m_s\Big)g\cos\phi\Big] + \Big(J_d' + m_s l_b^2\Big)\Big[\Big(R_s + m_s l_b l_s \sin(\phi-\theta)\dot{\phi}\Big)\dot{\phi}$$
$$-R_s\dot{\theta} - l_s m_s g\cos\theta\Big]\Big\}/\,\mathrm{Det} \quad (2)$$

where

$$\mathrm{Det} = \Big(J_d' + m_s l_b^2\Big)\Big(J_s + m_s l_s^2\Big) - \Big(m_s l_b l_s \cos(\phi-\theta)\Big)^2 \quad (3)$$

and

$$R_d' = R_d + R_m/r_g^2 \quad (4)$$
$$J_d' = J_d + J_m/r_g^2 + m_d l_d^2 \quad (5)$$
$$M_d = I_m r_m/r_g = (u_c r_c + I_{os})r_m/r_g \quad (6)$$

where, I_m is the motor current, u_c is the output value from the computer and I_{os} is the offset current.

For simulation model, the above equations are linearized at $\phi = -\pi/2$ and $\theta = \pi/2$.

2.3 Software of the System

The setup is connected with an IBM-PC compatible in which Turbo-C is used. The host computer is then connected with transputer networks in which OCCAM2 is used. After a C-program has called the function that boots the transputer, both programs are running at the same time, synchronizing each cycle. The cycle time is 10 msec. for both in simulation and in the real system. This can be achieved by the fast parallel calculation using transputers.

3. FINDINGS FROM MANUAL CONTROLLER ETC.

The several controllers were tested before designing the fuzzy controller. Statefeedback controller was designed based on the linearized equation of the system. The controller works well for the linearized point, but if, for example, the balanced angle of driving stick changed from 0 to a larger value, it cannot maintain its superior behaviour any more.

3.1 Manual Controller

To extract human behaviour for such a case, a manual controller is also added to the system. It consists of an analog dial which creates the command voltage for the motor torque directly. In the real system, however, it is very difficult to control the stick because of its fast time constant. We have introduced the delayed time constant system in the simulation by changing the moment of inertia of the balanced stick. A graphic monitor directly connected to a transputer displays the animated configuration of the system in real time, so the human can control the simulation system watching the monitor as well as the monitor display of the host computer on which time history of each variable can be seen.

When 10 times value of the original moment of inertia is adopted, it seems best for the human beings to control. After several trials, each operator will get his strategy to keep the stick upright. Some persons will try to keep the angle of the balanced stick as small as possible. But, after the stick begins to decline, he cannot stop the movement by maximum counter torque(Fig. 5). The other strategy is more effective. It is first initiated by a small swing of the driving stick(Fig. 6(a)). Then the balanced stick begins to decline to the other side. He will give the counter torque until the driving stick swings to the other side. Here the important point is to keep the declination of the balanced stick less than that of the driving stick(Fig. 6(b)). If the configuration of the system is kept as Fig. 6(b), he can maintain the system stable. In the case of human control, however, he cannot continue this configuration so long. If the configuration changes just as Fig. 6(c), he cannot keep it any more. This situation is same as Fig. 5(b). This

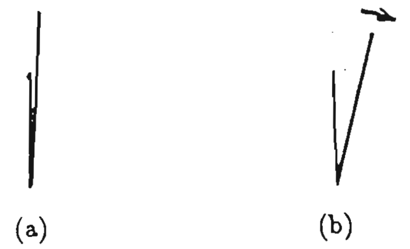


Fig. 5 Manual control strategy (1)

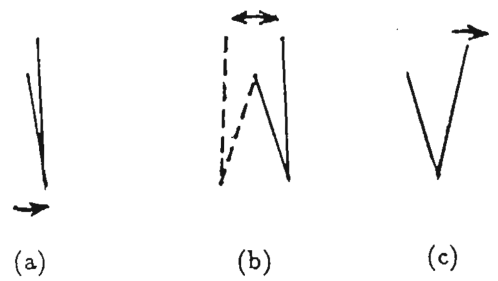


Fig. 6 Manual control strategy (2)

means that if the only angle of balanced stick is taken into account, the controller cannot succeed.

3.2 Play Back Monitoring

As well as manual controller, play back monitoring and phase plane trajectory are used to analyze the behaviour of each controller. Play back monitoring is the replay facility of the previous trials. After each trial data are saved into a data file. The play back monitoring will accept this file as the data and show each variable as well as the animated configuration. Besides, we can stop the monitoring and restart it any time we want. By analyzing the failed trials of several controllers, the final stages were always similar to the configuration as shown in Fig. 6(c).

4. DESIGN AND RESULTS OF FUZZY CONTROLLER

4.1 Design of Two-input One-output Fuzzy Controller

The fuzzy controller was designed for this setup, taking account of the results of manual control and other controllers. There are several stages to build up the fuzzy control rules and the definition of the membership functions of both inputs and output. First, the system is divided into two parts; the control of the balanced stick and the control of the driving stick. It is already found that there are interaction between these two parts and in the statefeedback controller these interaction is taken into account, in fact. However, it is still worthy to know how the controller works.

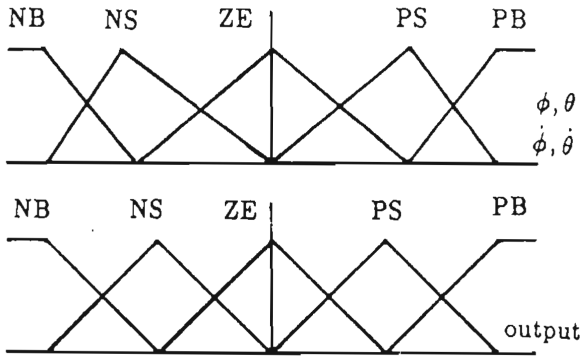


Fig. 7 Definition of membership functions

Table 1 Fuzzy control rules

$\dot{\theta}, \dot{\phi}$ \ θ, ϕ	NB	NS	ZE	PS	PB
NB	PB	PS	PS	ZE	ZE
NS	PS	PS	PS	ZE	ZE
ZE	PS	PS	ZE	NS	NS
PS	ZE	ZE	NS	NS	NS
PB	ZE	ZE	NS	NS	NB

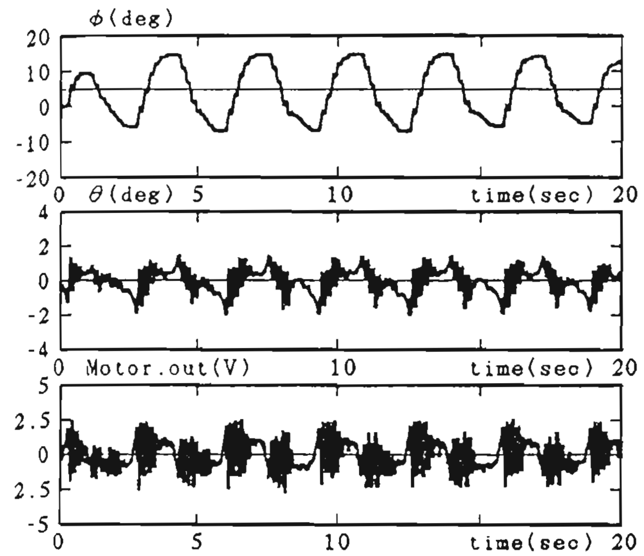
The membership functions are defined as shown in Fig. 7 and the control rules are shown in Table 1. The control table is applied to both parts. The two outputs are added with weights and applied to the motor.

4.2 Results of Two-input One-output Fuzzy Controller

A sample result of the fuzzy controller is shown in Fig. 8. This is the case for $\phi = 4$ deg. It works stable for over several hours. It is also found that this controller is stable for impulsive and constant disturbances applied by a finger up to a certain amplitude.

4.3 Design of Four-input One-output Fuzzy Controller

If the disturbances applied to the two-input one-output fuzzy controller is big enough, the controller fails to keep the stick upright. This is caused by the fact that the states is not independent between the angles of the driving stick and the balanced stick. So, to improve these situation,



. 8 A sample result of two-input one-output fuzzy controller

four-input one-output fuzzy controller is designed. The inputs are $\phi, \theta, \dot{\phi}$ and $\dot{\theta}$. The output is the voltage to the motor. However, it is difficult to build the control rules. Besides, the number of control rules becomes quite large. We have made one example of this type of fuzzy control rules, but it is not tested.

4.4 Towards a Learning Controller

There are already proposed several methods to implement learning capability for fuzzy controller. The combination with artificial neural networks is most promising. There are two points where learning capability should be considered; definition of membership functions and fuzzy rules. For these learning Hayashi *et al.* [4] introduced artificial neural network-driven fuzzy control (NN-driven fuzzy control) using the cart-pole system. The initial data 's taken from the human behaviour. They used only two .uzzy rules to balance the pole from the hanged-down position. This approach is under considering to replace with the four-input one-output fuzzy controller.

5. CONCLUSIONS

The control of an inverted pendulum was done using several control methods, though the results are only shown for the fuzzy controller. The main conclusions obtained are summarized as follows:

1. Fuzzy controller initially designed by try-and-error method works satisfactory for the inverted pendulum.
2. The difficult situation of its control was analyzed by manual control and play back monitoring of other control methods.
3. The learning capability for nonlinear system was discussed and the combination of fuzzy control and artificial neural networks is to be the most possible solution for this problem.

The following points are the recommendation for the future study.

1. Application of NN-driven fuzzy control to this setup
2. Learning capability for the change of the system environment
3. Possibility of a neural controller without teaching data

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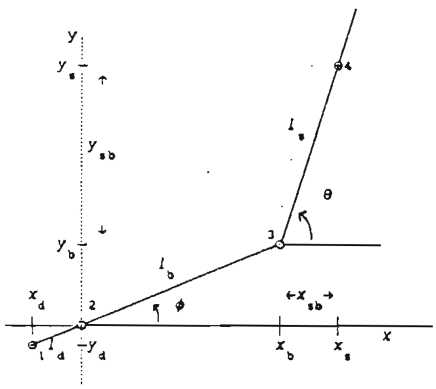

Fig. A1 Definition of coordinate base and angles ϕ and θ

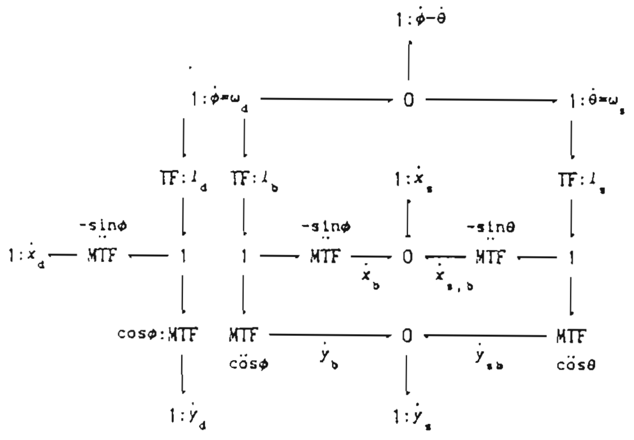

Fig. A2 Junction structure of balanced stick system

APPENDIX

A.1 Bond Graph Description of the System

By referring the definition of coordinate base shown in Fig. A1, junction structure is made as in Fig. A2. In this figure the upper three 1–junctions represent angular velocities. The four 1–junctions marked with $\dot{x}$ or $\dot{y}$ represent the translational velocities of the centres of mass of driving and balanced sticks in x and y direction respectively. The three 1–junctions between TF's and MTF's are inserted mainly because they reduce the number of calculations. They can be thought of as the magnitude of the velocity of the centre of mass of driving stick, the velocity of the hinge point or balanced stick axis and the velocity of the centre of mass of the balanced stick relative to the balanced stick axis respectively.

In this junction structure the elements representing dynamic properties should be added. Connected to the 1–junction of $\dot{\phi}$ is the motor torque M_d, friction R_d and inertial moment J_d. There is friction R_s on the balanced stick axis, which is connected to the 1–junction of the difference of two angular velocities. The inertial moment of the balanced stick J_s is connected to the 1–junction of the angular velocity $\dot{\theta}$. The masses m_d of the driving stick and m_s of the balanced stick are connected to the 1–junctions representing the translational velocities in x and y direction respectively. The effort sources representing the gravity force, $-m_d g$ and $-m_s g$, are connected the gravity force working in the negative y direction. The motor is modelled as a gyrator with ratio r_m. On the motor axis there is a friction R_m and an inertial moment J_m. The gear has a ratio r_g.

A.2 Causality

A causal conflict would arise if all inertances had integral causality. It is also preferred to have as little inertances with derivative causality as possible, as in numerical simulation differentiating is less accurate than integrating. In combination with an inertance with integral causality it may even cause numeric instability.

A.3 Estimates of Parameters

The estimates of the parameters, obtained by system identification, are listed below:

r_c	=	4.95	±	0.05	A
I_{os}	=	−0.02	±	0.05	A (neglected)
R_{nl}	=	0.2	±	0.1	Nm
r_m/r_g	=	0.57	±	0.05	NmA^{-1}
$l_d m_d$	=	−0.005	±	0.005	kgm
J'_d	=	0.07	±	0.02	kgm^2
R'_d	=	0.05	±	0.1	Nms
l_s	=	0.165	±	0.005	m
m_s	=	0.15	±	0.01	kg
J_s	=	0.0023	±	0.0002	kgm^2
R_s	=	0.45	±	$0.05 \cdot 10^{-3}$	Nms
l_b	=	0.20	±	0.01	m

where, R_{nl} is the Coulomb friction acting on the driving stick.