AUTOMATIC NAVIGATOR-INCLUDED SIMULATION FOR NARROW AND CONGESTED WATERWAYS

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1. ABSTRACT

A method of fast-time simulation including a navigator's model is proposed. This method is an application of Ship Auto-navigation Fuzzy Expert System (**SAFES**) and in this paper some considerations and modifications necessary for applying it to harbour and waterway designs are described. An example actually used for a certain bypass design of a particular congested waterway is shown and it is verified the system is useful for these applications.

2. INTRODUCTION

When a new harbour or a waterway is designed, it is, of course, necessary to take care of the natural configuration and environmental conditons as well as the particulars of the considering ships. In straight waterways or waterways with low traffic density, however, some simple formulae constructed mainly by principal dimensions are usually used to determine the waterway width[1]. In bent waterways or waterways with frequent traffic, we should further take care of more detailed manoeuvring performances of the considering ship, human behaviours, and even motions and behaviours of other traffic vessels.

For this purpose, safety assessment using a ship handling simulator plays an important role. The problem in this case lies rather on the psychological effects concerning to the visual display etc. and the experiment costs. Furthermore, the traffic effect is usually difficult to implement, although the author and others have proposed a methodology realizing intelligent movements of target ships in a ship handling simulator[2].

Thus, the demand is growing for fast-time simulation with more realistic navigator's model to assess safety navigation of new harbours and waterways, or to select suitable scenarios for real-time simulator exeperiments. In the fast-time simulation technique, the mathematical model of the ship dynamics in various environmental conditions should be of course carefully investgated, but the modelling of a navigator is another important factor especially for narrow and congested waterways. On the other hand, automation in ship navigation is coming to new age of "intelligent ships". Here, the key point of intelligent ships is again how to represent and integrate navigator's knowledge and experiences.

The author has already proposed a basic model of navigators necessary for from usual navigation to collision avoidance[3] and an expert system approach to collision avoidance in multi-ship encounters has been also proposed by Koyama *et al.*[4]. In the system called **SAFES**(Ship Auto-navigation Fuzzy Expert System)[5], both are combined and revised more suitable for applications to narrow and congested waterways. There are already two actual applications using **SAFES**. One is realistic simulation of marine traffic flow in the one of the most congested waterways in Japan, where over 600 vessels in a day are passing and crossing in a #-shape waterway. The other is done to design a bypass waterway, even which is very narrow and bent, and has strong current. In the paper, the micro simulation technique including navigator's skill is to be described with full reference to the latter application.

3. SHIP AUTO-NAVIGATION FUZZY EXPERT SYSSTEM (SAFES)

The system itself[5] is an expert system written in OPS83. In the system, the basic subsystem is the modelling of behaviours of a navigator and a helmsman including collision avoidance with one target ship[3]. Though the subsystem was converted to C from FORTRAN77 and if–then type rules especially for collision avoidance were extracted to implement into an expert system, there are only a few modifications from the original version of a navigator's model[3]. In this section, the system will be described briefly with explanation of revised or modified points.

3.1 Modelling of a navigator

The following items are all modelled using fuzzy theory.

- **Fear of collision** is reasoned from TCPA (Time to the Closest Point of Approach) and DCPA (Distance of the Closest Point of Approach) as same as the original version[3], but the maximum values of the membership functions are modified so as not to *feel* the fear of collisions in normal sailing condition within the given waterway.
- **Path keeping.** The course changing point is assumed as a buoy and the nearness to the buoy is reasoned just as the analogy to the fear of collision. The maximum values of the membership functions are also modified to match the waterway width and curvature. According to the nearness to the course changing point the course command is reasoned from the course to the course changing point and the course to the next course changing point. This controller is especially useful for narrow and bent waterway navigation, because we need only input some points on course changing points, but not the points of starting the course changing. The controller automatically order the necessary rudder commands at proper timing, even if the ship is sailing off the designed path. Figure 1 shows an example of input points for path keeping.

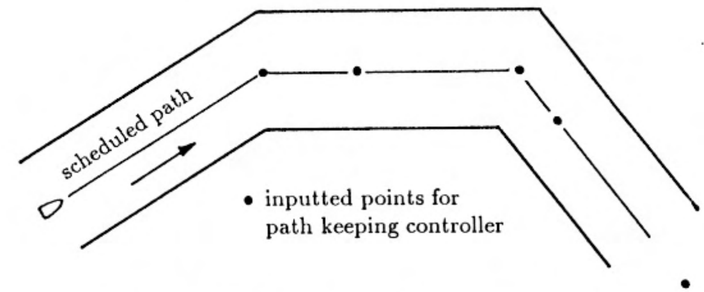

Figure 1. Example inputs for path keeping controller.

- **Course keeping/changing.** From the inputs of heading error and rate of turn, the output of rudder angle is reasoned. This fuzzy controller — called *fuzzy autopilot* — is superior in smooth change of its control gain from course keeping to course changing.
- **Waterway boundary detection** is a new feature provided for narrow and bent waterways. As shown in Figure 2, false-ship concept is introduced to detect the boundary. In Figure 2 (a) false-ships are placed on the crossing point of the nearer boundary with heading course and on the boundary aside of the own ship respectively. In the case as shown in Figure 2 (b), only the side false-ship is placed. After placing false-ship(s), we no longer take care of the boundary itself, and false-ship(s) are regarded as usual ships execpt the following limitations.
  - The false-ships are created, when there are crossing points between the *true* ship and the boundary.
  - Otherwise, the false-ships are removed from the working memory.
  - The false-ships move only along the boundary according to the movement of the accompanying *true* ship.
  - The false-ships themselves take no action for collision avoidance with the accompanying ship, nor other ships including other false-ships.

The false-ships themselves have no *fuzzy* effect, but the fear of collision to the boundary is again reasoned *fuzzy* using the false-ships.

Calculation of crossing points including parallel condition between a given point (the *true* ship) and a polygon (the waterway boundary) is unexpectedly troublesome. A set of subroutines to handle picture interference as shown in Table 1[6] is useful for this purpose, because it can even check if any part of the ship outline touches the boundary or if the ship is inside the waterway.

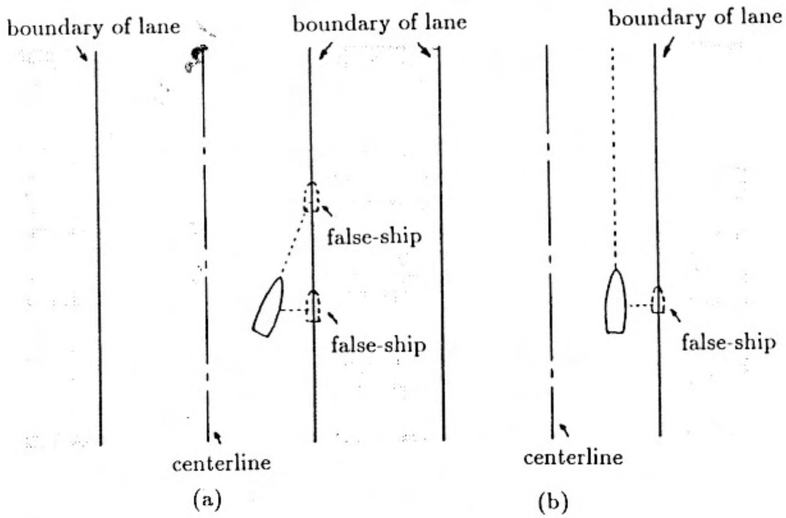

Figure 2. False-ship concept to detect waterway boundary.

Table 1. List of functions for picture interference perception[6].

Name	Function
line-length	to calculate length between two points
line-crossing-point	to calculate a crossing point of two lines
line-crossing-angle	to calculate angle between two vectors
check-point-on-line	to check whether a point is on a line segment or not
check-line-parallel	to check whether two lines are parallel or not
check-line-cross	to check whether two line segments are crossing or not
check-line-touch	to check whether two line segments are touching or not
check-point-on-polygon	to check whether a point is on an edge of a polygon or not
check-point-include	to check whether a point is included in a polygon or not
check-polygon-cross	to check whether two polygons are crossing or not
check-polygon-touch	to check whether two polygons are touching or not
check-polygon-include	to check whether a polygon is included in the other or not
check-interfere	to calculate *spatial distance* between two polygons

3.2 Expert system approach

Even if elemental models of a navigator are obtained for various navigational situations, it is not convenient to describe all possible combinations of situations using conventional programming language. Expert system is suitable for these applications and OPS83 is one of the tools to describe an expert system.

a. Working memory configuration. In OPS83, the working memory configuration or definition of *elements* is important. In the system **SAFES**, the following elements are defined.

- **ship** element expressing each ship with information of
  - physical features
  - state of navigation
  - state of motion
  - state of avoiding action plan.
- **target** element being created if any two ships *feel* the fear of collision and being removed if the fear disappers. It contains information about
  - own ship's number and other necessary states
  - target ship's number and other necessary states
  - fear of collision.
- **if-target** element being used for determining the timing of returning to the original path during avoiding. It contains
  - own ship's number
  - target ship's number
  - fear of collision, if the own ship now changes the course directing to the original path.
- **obstacle** element indicating the boundary of a waterway
- **start** and **goal** elements being used for the process control

b. Knowledge base. Knowledge base is a collection of rules which contains

- rules for process control
- rules for simulation control including graphic display
- rules extracted from regulations such as International Regulation for Preventing Collision at Sea

- rules extracted from a navigator's model
- other implicit rules such as
  - "Do not overtake a ship which is overtaking, but reduce speed".
  - ...

c. Graphic display. Graphic display is a good interface showing the system status and the results. In the system **SAFES**, graphic windows are divided as follows:

- **absolute plot window** which shows the bird's eye view with the traced positions of each ship
- **relative plot window** which shows the traced radar's view of a particular ship specified by an operator
- **perspective view window** which shows the perspective view from the bridge of the specified ship
- **console window** which shows various information such as the present rudder angle of the specified ship or the fear of collision between a certain ship etc.

The graphic displays are created by setting *cameras* suitable for each window and the graphic data itself is unique in the system. Recent graphic workstation (GWS) has such capability with standard graphic subroutines. An example of multi-ship encounter is shown in Figure 3, where absolute plot and four relative plots windows are used.

4. APPLICATIONS OF SAFES

As **SAFES** itself is a navigator's model in multiple ship environment and contains simulation capability, it can be applicable to various problems such as

- fast-time simulation to
  - design or assess narrow and bent waterway navigation
  - provide scenarios for ship handling simulator experiments
- generation of realistic traffic environment to
  - evaluate of an automatic navigation system or a vessel traffic control system
  - provide visual and radar background of a ship handling simulator[2]
  - evaluate or predict traffic environment itself for waterway design[7].

In this section, two of them carried out for actual projects will be introduced to show the effectiveness of **SAFES** and the future problems to be revised.

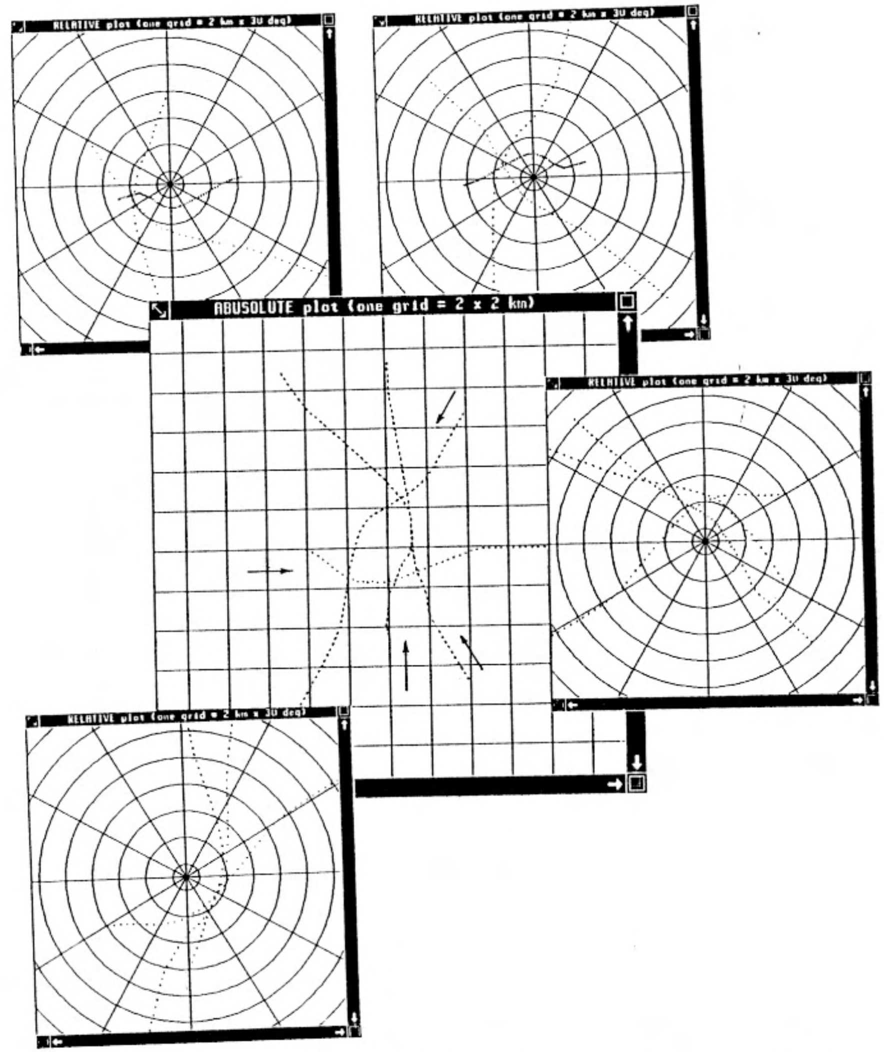

Figure 3. An example of four-ship encounter at the origin by **SAFES**.

Table 2. Principal particulars of type ships used for simulation.

Item	999GT-type Ship	499GT-type Ship
L_{PP}(m)	75.0	62.0
B_m (m)	12.7	10.0
d_m (m)	4.9	4.17
trim(m)	1.0	0.0
C_B	0.72	0.70

4.1 Assessment of a narrow waterway design

Inland Sea area of Japan (cf. Figure 4) is the important route for product carriers including chemical tankers and car carriers. This area is also very important fishing field. At the same time, it is also famous for its beautiful scenery with many small islands and nowadays there are already many long bridges and plans connecting the main island to small islands or even to *Shikoku* island.

The small islands prevent the straight waterway configuration nor enough sectional area in both width and depth. Besides, there are normally strong current and in spring sometimes heavy fog. So, there happens frequent casualties. Especially, *Kurushima Straits* is notorious for its narrow and poor outlook with strong current of 8 knots maximum. Of course, various countermeasures are provided such as a unique regulation "to sail west lane when the current is coming and to sail middle lane when the current is going" as well as a current signal, but growing of sea traffic makes some unfortunate encounters of ships.

So, the next countermeasure is planed to construct a bypass waterway for smaller vessels to reduce the main traffic. However, this bypass itself is still narrow and bent as roughly shown in Figure 5 with strong current and several rocks. Therefore this route called *Miyanokubo Straits* was naturally used only by local fishermen or local cargo shipmasters. The plan is to consolidate the waterway to allow sailing up to 1,000GT vessels with navigational aids and dredging, if necessary.

Simulation conditions. In a committee to assess the safety and to plan the waterway configuration, the author has carried out the simulation using **SAFES** for some alternative waterway designs. The simulation was done concerning to the following items using a 999GT-type cargo ship and a 499GT-type cargo ship whose particulars are shown in Table 2.

- Is proposing two-way traffic route using *Funaore Strait** as shown in Figure 6 safe for a 1,000GT cargo ship meeting with a 500GT cargo ship? Here, the problems are as follows.

* *Funa-ore* is read *Ship-broken* in Japanese.

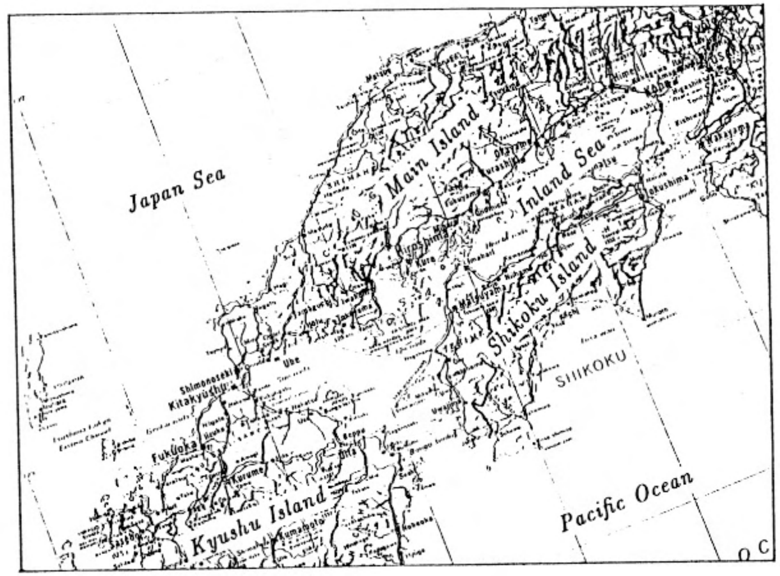

Figure 4. *Inland Sea* area of Japan.
(from Britanica International Atlas)

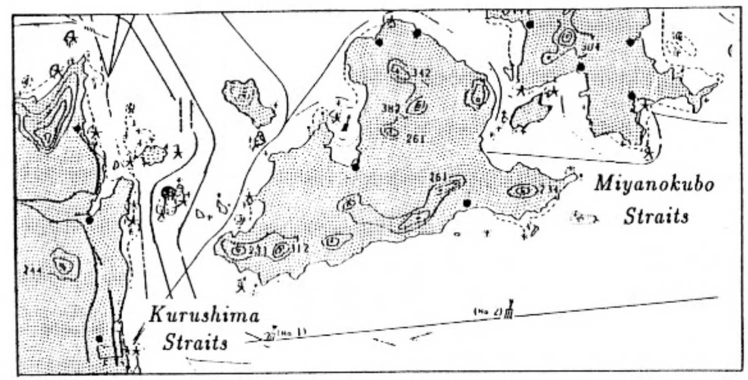

Figure 5. *Kurushima Straits* and its bypass *Miyanokubo Straits*.

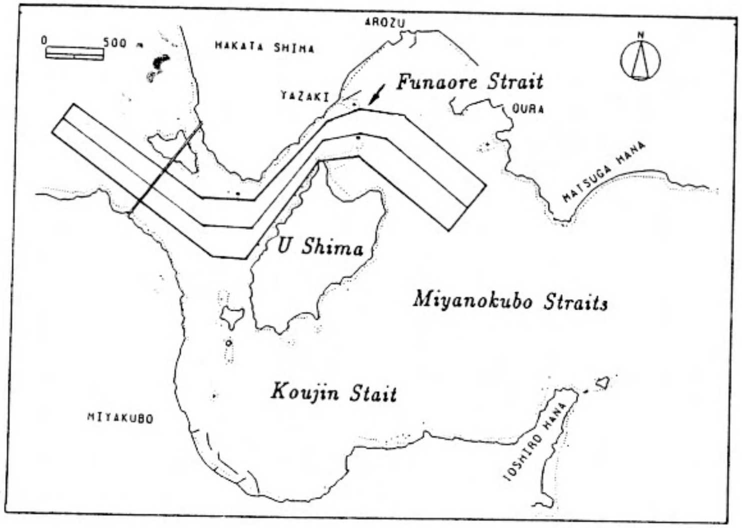

Figure 6. A proposed two-way traffic route set in *Funaore Strait* of *Miyanokubo Straits.*

- The width at the most narrow part of the waterway is only 160 m, where the curvature of the waterway is also worst.
- Several rocks and a local harbour area are adjacent, so it is very difficult to widen the curved area.
- The strong current exists and islands prevent long-distant view.

- If the answer of the above question is no, is it possible to propose any one-way separated traffic route using *Koujin Strait*† west- and southward of an Island *U Shima*? In this case, still the following points should be considered.
  - There are many rocks in the westward of *U Shima.*
  - The southward of *U Shima* is especially a nice fishing field.

<u>**b. Current effect.**</u> As for the mathematical model of ship motion in non-uniform current and estimation of its derivatives etc., several models and methods are already proposed (*cf.*Appendix), and it seems to be necessary to simulate the current effect. Indeed, according to the *questionaire* carried out to investigate the traffic problems in *Miyanokubo*

† *Miyanokubo Straits* are the generic name including *Funaore Strait* and *Koujin Strait.*

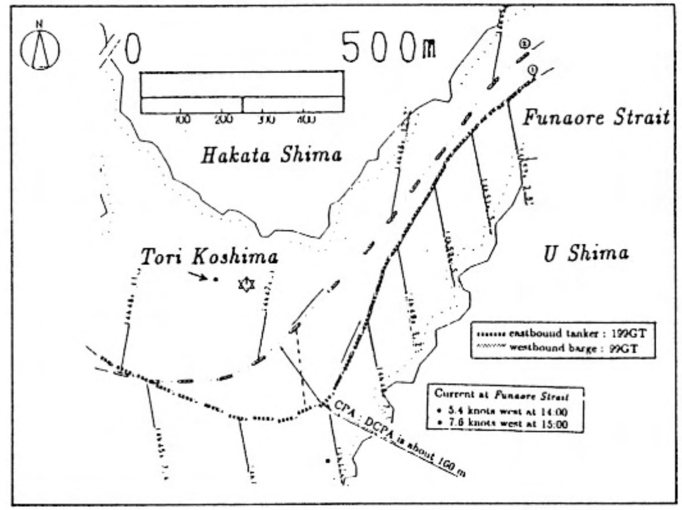

Figure 7. An example of actual ships' path in *Funaore Strait.*

Straits[8], 14.3 % of repliers of shipmasters pointed out the strong current is one of the navigational difficulties in this area. Therefore, in most cases except the cases where the arrival time is rigid, shipmasters answered that they operate their ships when the current is same direction with their ships' direction or weak enough not to be affected.

In the actual ship path investigation using a fixed radar on land[8], we cannot find significant effects by current, although the investigation was carried out at nearly the strongest current. Figure 7 is one of the exceptional cases in which we can imagine there was slight current effects on the eastbound tanker. This may mean that the current effect itself is very big, but that each helmsman compensates its effect within his control loop. We can find the interesting results in the simulator study carried out for *Kurushima Straits*[9] concerning to the human ability of compensating the current effect and its effect during the simulator experiments for various types of ships with different stability. The author would like to point out that in the fast-time simulation we *may* ignore the current effect to some extent and regard it as relative ship speed. Of course, in ship handling simulator experiments, we cannot neglect the non-uniform current dynamics and the current effect in fast-time simulation should be carefully investigated in the future work.

Furthermore, in this case, no detail data of captive model tests etc. is available and the considering ships are rather small. So, as a first step of research, Nomoto's so-called K–T model[10] was used for each ship's motion, while the coefficients were estimated through the procedure describe in **Appendix**. The water depth is assumed to be dredged constant of 7.35 m thoughout the waterway. No bank effect nor approaching ship's interaction is considered. As for the current and the constant wind will be planed to be implemented in the mathematical model, though they are neglected here.

c. Validation of the simulation. It is necessary to check if the simulation results are creditable or not. Validation itself is rather difficult, but here rough validation was done to tune the navigator's model so as to navigate successful in the given waterway. Figure 8 is one of the validation result where westbound 999GT-type ship and eastbound 499GT-type ship are designed to sail at the centerline of the waterway to meet at the west part of the waterway. The number of "course changing points" given to each ship's "modelled navigator" is around 5~8. Each ship-mark is the real size and direction and plotted every 1 minute. The long arrows show each ship's sailing direction and the small numbers aside the ship-marks indicate the time from the simulation start time. The bidirectional arrow connecting each ship is the DCPA (Distance of the Closest Point of Approach). Each grid is plotted every 0.5 km and the proposed waterway boundary is indicated by two polygons as shown in the figure.

Each ship could sail fairly smooth to trace the centerline of the waterway, and successfully avoid a collision at around 22 mintues. Ship speed is 5 knots each. In this case, the eastbound ship sailed too close to the waterway boundary there, but in the actual navigation both ships will sail right-hand side of the waterway as shown in the following figures. So the basic features of **SAFES** can be regarded to be applicable to such a waterway navigation.

d. Evaluation of two-way traffic. Various situations in this route will be discussed by simulation results. Figure 9 is the same case with Figure 8, but each ship is sailing in right-hand part of the waterway (case 1.2). In this case the westbound ship slightly avoided at around 22 minutes and both ships could pass the waterway safely. Following two cases show the effect of meeting place. All the conditions except the meeting place are same with case 1.2. In Figure 10 (case 1.3) both ships could pass in the narrowest part of the waterway, but there are not enough distance between the boundary. In Figure 11 (case 1.4), where the condition is more severer, it is rather difficult to keep safe distance between the boundary and the eastbound ship overturned after the collision avoidance.

The following two cases are the same with case 1.3 except the ships' speed. In Figure 12 (case 1.3U6) the speed of both ships is set 6 knots and in Figure 13 (case 1.3U7) it is set 7 knots. The faster the ship speed is, the greater is the degree of the overturn. One of the reason for this is the reasoning interval. In the system **SAFES** ship motion calculation is carried out every 1 second, but the navigator model judges and take orders every 10 seconds. We should change the reasoning interval according to the size of the ship or the ship speed. Several other simulations with different conditions were also carried out, and the author has concluded that there may happen dangerous cases, if this route will be allowed two-way traffic.

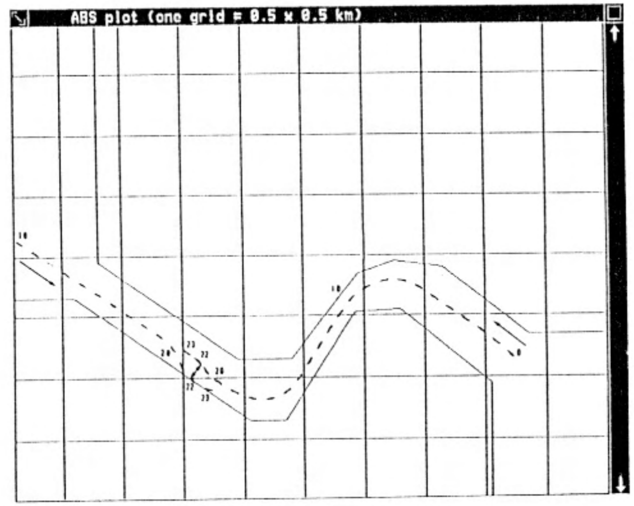

Figure 8. Two-way traffic route (case 1.1).

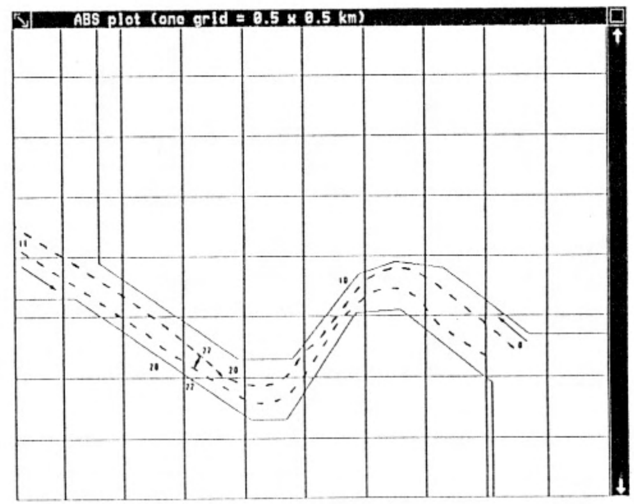

Figure 9. Simulation result of two-way traffic route (case 1.2).

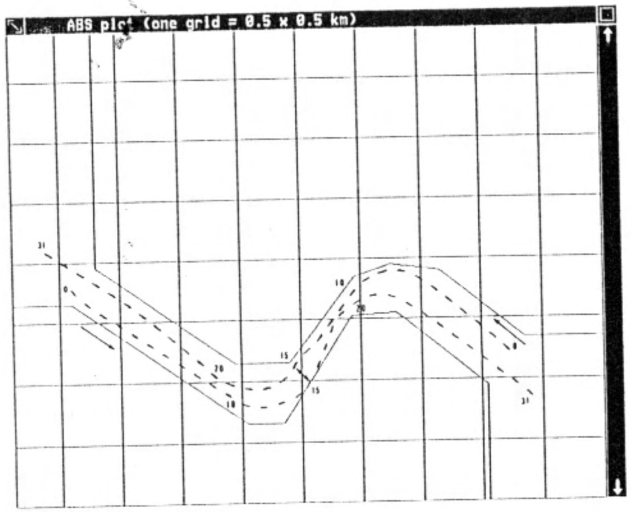

Figure 10. Simulation result of two-way traffic route (case 1.3).

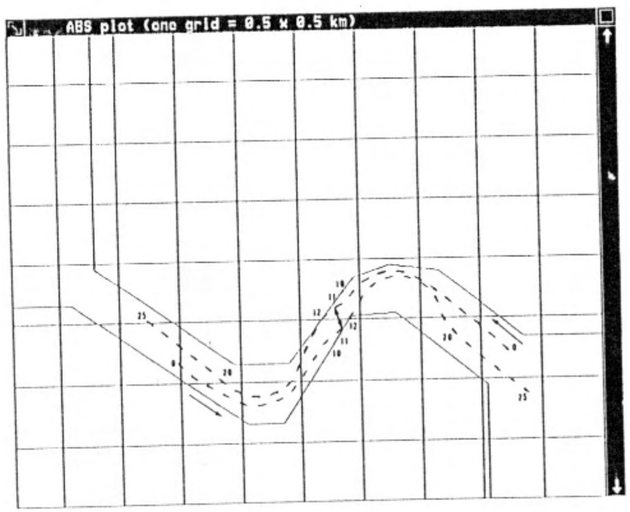

Figure 11. Simulation result of two-way traffic route (case 1.4).

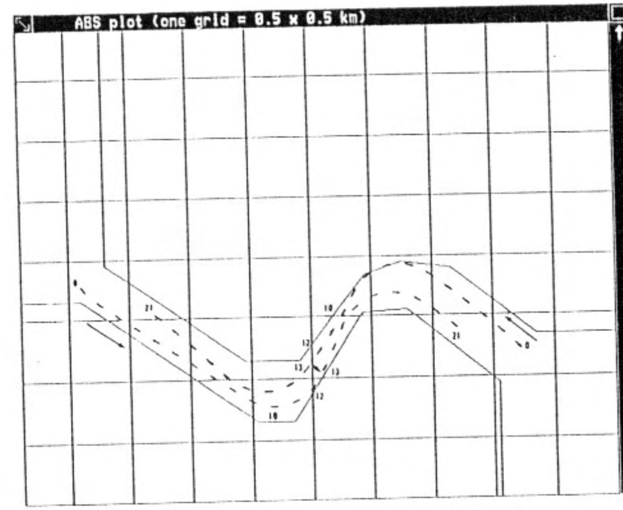

Figure 12. Simulation result of two-way traffic route (case 1.3U6).

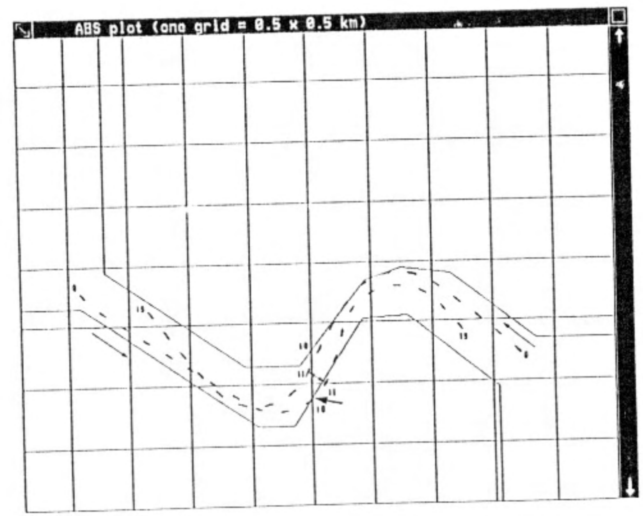

Figure 13. Simulation result of two-way traffic route (case 1.3U7).

e. Evaluation of one-way traffic. Next, one-way separated waterway was evaluated using **SAFES**. In this waterway, the westbound ships will sail north part of the route and the eastbound ships will sail from west part to the south part of the separated route. It is verified in the previous one-way traffic simulation that in the west part of the route, it is not so difficult to sail safely, even if two ships will meet there. So, in this section the following two points will be checked.

- Is it possible to turn 90 degrees at the southern route?
- Is it possible to permit overtaking actions for both routes?

In Figure 14 (case 2.1), the former point is checked. In the figure the westbound ship is the 999GT-type ship at 5 knots and the eastbound ship is the 499GT-type ship at 5 knots. In the following two figures, the latter point is checked. In Figure 15 (case 2.2), the 999GT-type ship at 5 knots will be overtook by the 499GT-type ship at 6 knots in the westbound and the 499GT-type ship at 5 knots will be overtook by a 199GT-type ship[†] at 7 knots. Figure 16 (case 2.3) is the same condition with Figure 15 except that the overtaking places are more serever. In the case of Figure 16, both overtakings are quite dangerous. The near-miss point or out-of-bound point are indicated by short arrows. Of course, the actual pilots may navigate safely even in these cases, but these simulation shows the possibility of the casualties and they can be used to evaluate the waterway configuration.

4.2 Marine traffic evaluation

There is already a method to evaluate marine traffic statistically, which is so-called macro or network simulation[11]. This method is an application of queucing theory and can evaluate the traffic from quantitative aspect, so it is widely used for various projects designing or assessing waterways in Japan.

However, it cannot deal with precise manoeuvring properties of each ship according to each navigator's behaviours such as collision avoidance manoeuvres. Micro simulation[12] is thus proposed to fill these features into macro simulation. The author has also applied **SAFES** to micro simulation of a #-shape waterway[7]. Figure 17 is an example of the result of this simulation, where over 600 vessels with statistical speed, size and manouevring properties distributions were generated at statistical interval of arrival time in 24 hours. We can roughly evaluate the navigational safety by analyzing the results.

[†] The particular of this ship is $L_{PP} \times B_m \times d_m = 50.0 \times 10.0 \times 6.0$ (m).

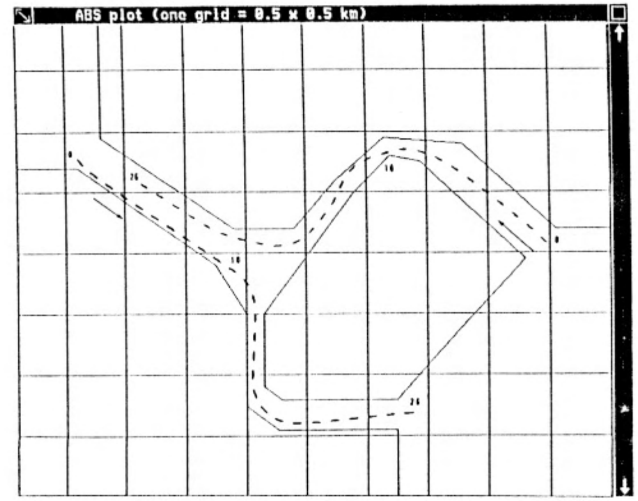

Figure 14. Simulation result of one-way traffic route (case 2.1).

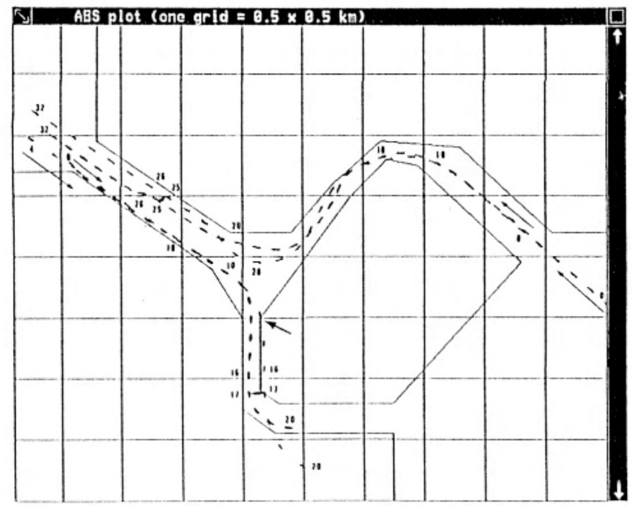

Figure 15. Simulation result of one-way traffic route (case 2.2).

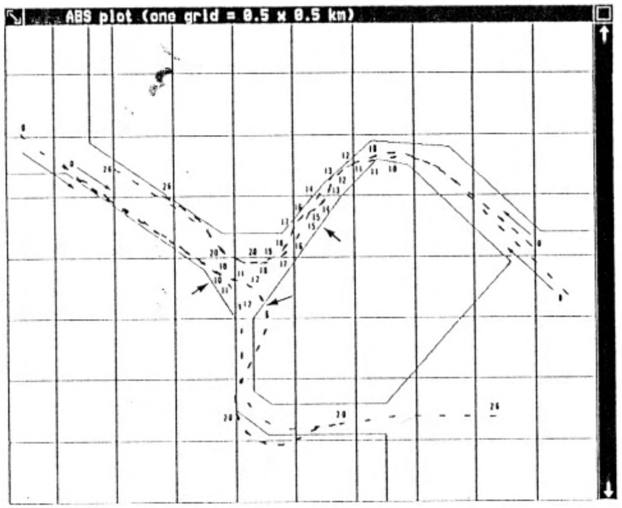

Figure 16. Simulation result of one-way traffic route (case 2.3).

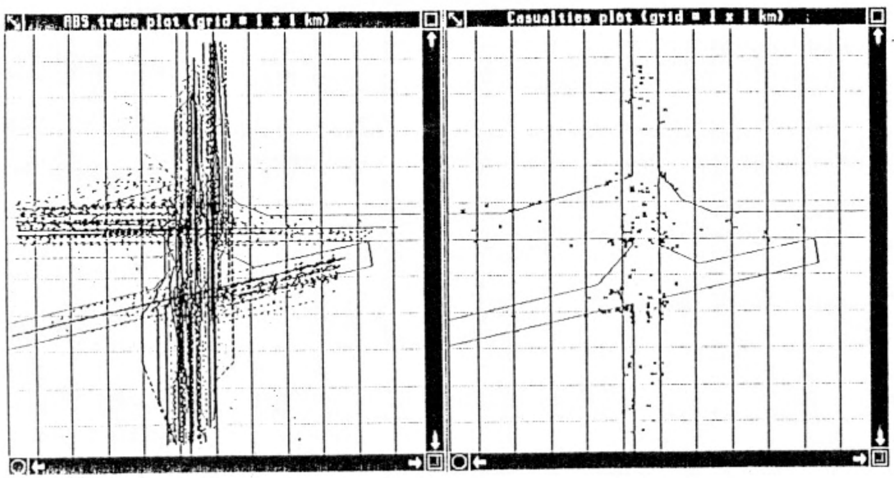

Figure 17. An example of marine traffic simulation using **SAFES**.

5. CONCLUDING RENARKS

In this paper, a method of modelling of a navigator and its integration to a fast-time simulation method **SAFES** are introduced and the validity of these methods are shown through two actual applications. There still remain many problems to be solved, and the author would like to further investigate this methodology for more practical applications especially for the waterways design.

The main conclusions obtained through this research are summerized as follows.

- The model of a navigator combining fuzzy theory and expert system is useful for integrating into a fast-time simulation system.
- The fast-time simulation system called **SAFES** is effective for various applications, especially for assessment of narrow and congested waterways.

The following points should be further improved to make the system more reliable and useful.

- The mathematical model of ship motions for various environmental conditions
- Constructing more reliable knowledge base including more natural and common-sense knowledge
- Improving the structure of knowledge base for reducing reasoning and judging time
- Easy handling for setting the simulation parameters
- Establishing statistic data acquisition and qantitative analysis system

ACKOWLEDGEMENTS

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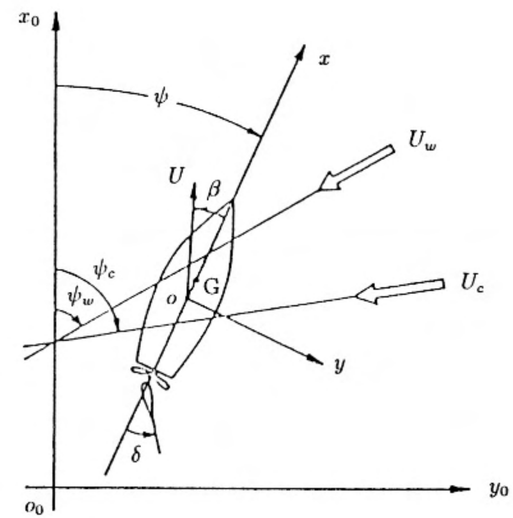

Figure 18. Coordinate system.

APPENDIX MATHEMATICAL MODEL OF SHIP MOTIONS

A.1 Model of ship mations in non-uniform current

The equations of ship motions in non-uniform current[A.1] may be expressed as follows according to the coordinate system shown in Figure 18.

$$\begin{aligned}(m+m_y)\dot{u} &= (m+m_y-X_{vr})v_a r+(mx_G+X_{rr})r^2 \\ &\quad +X_{vv}v^2-(m+m_y)U_c r\sin(\psi_c-\psi) \\ &\quad +(1-t)T-R_s-(1-t_R)F_N\sin\delta+X_w \qquad (1)\end{aligned}$$

$$\begin{aligned}(m+m_y)\dot{v}-(Y_{\dot{r}}-mx_G)\dot{r} &= Y_v v_a+(Y_r-mu_a)r \\ &\quad +Y_{NL}(v_a,r)+(m+m_y)U_c r\cos(\psi_c-\psi) \\ &\quad -(1+a_H)F_N\cos\delta+Y_w \qquad (2)\end{aligned}$$

$$\begin{aligned}(I_z+mx_G^2+J_z)\dot{r}-(N_{\dot{v}}-mx_G)\dot{v} &= N_v v_a+(N_r-mx_G u_a)r \\ &\quad +N_{NL}(v_a,r)+(mx_G-N_{\dot{v}})U_c r\cos(\psi_c-\psi) \\ &\quad -(x_R+a_H x_H)F_N\cos\delta+N_w \qquad (3)\end{aligned}$$

where
m : mass of a ship

m_x : longitudinal added mass of a ship
m_y : transverse added mass of a ship
I_z : moment of inertia around z-axis of a ship
J_z : added moment of inertia around z-axis of a ship
r : rate of turn around z-axis of a ship
x_G : x-ordinate of center of gravity of a ship
F_N : nominal force acting on a rudder
X_w : longitudinal force acting on a ship by wind
Y_w : transverse force acting on a ship by wind
N_w : moment around z-axis acting on a ship by wind
u_a, v_a : longitudinal and transverse relative flow velocities by current and can be expressed as

$$u_a = u + U_c \cos(\psi_0 - \psi) \quad (4)$$
$$v_a = v + U_c \sin(\psi_0 - \psi) \quad (5)$$

u : longitudinal component of ship velocity U
v : transverse component of ship velocity U
U_c : current velocity
Y_{NL}, N_{NL} : nonlinear hydrodynamic force (transverse) and moment (around z-axis) and they can be expressed as follows using cross-flow model.

$$Y_{NL}/\frac{\rho}{2}U_a^2 Ld \equiv Y'_{NL} = -C_D \int_{-1/2}^{1/2} |v'_a + \xi r'|(v'_a + \xi r')d\xi \quad (6)$$
$$N_{NL}/\frac{\rho}{2}U_a^2 L^2 d \equiv N'_{NL} = -C_D \int_{-1/2}^{1/2} |v'_a + \xi r'|(v'_a + \xi r')\xi d\xi \quad (7)$$

where
C_D: cross-flow drag coefficient

$$v'_a = v_a/U_a \quad (8)$$
$$r' = rL/U_a \quad (9)$$
$$\xi = x/L \quad (10)$$
$$U_a = \sqrt{u_a^2 + v_a^2} \quad (11)$$

A.2 Estimation of derivatives

If data of captive model tests etc. are not available, following formulae are useful to estimate the derivatives.

a. Linear derivatives. Inoue *et al.* have derived the following semi-empirical formulae[A.2].

$$Y'_v = -(\frac{\pi}{2}\Lambda + 1.4C_B\frac{B}{L})(1 + \frac{2}{3}\frac{\tau}{d_m}) \quad (12)$$
$$Y'_r = \frac{\pi}{4}\Lambda(1 + 0.80\frac{\tau}{d_m}) \quad (13)$$
$$N'_v = -\Lambda(1 - \frac{0.27}{l_\beta}\frac{\tau}{d_m}) \quad (14)$$
$$N'_r = -(0.54\Lambda - \Lambda^2)(1 + 0.30\frac{\tau}{d_m}) \quad (15)$$

where
τ : trim
and

$$\Lambda = \frac{2d_m}{L} \quad (16)$$
$$l_\beta = \Lambda/(-\pi\Lambda + 1.4C_B\frac{B}{L}) \quad (17)$$

b. Added mass and added moment of inertia. Motora's charts[A.3] are very reliable, but here it is convenient to use the regression forms proposed by Clarke *et al.*[A.4].

$$m_v/\frac{\rho}{2}L^2 d \equiv m'_y = \pi\frac{d}{L}\{1 + 0.16C_B\frac{B}{d} - 5.1(\frac{B}{L})^2\} \quad (18)$$
$$J_z/\frac{\rho}{2}L^4 d \equiv J'_z = \pi\frac{d}{L}\{\frac{1}{12} + 0.017C_B\frac{B}{d} - 0.33\frac{B}{L}\} \quad (19)$$
$$Y_{\dot r}/\frac{\rho}{2}L^3 d \equiv Y'_{\dot r} = -\pi\frac{d}{L}\{0.67\frac{B}{L} - 0.033(\frac{B}{d})^2\} \quad (20)$$
$$N_{\dot v}/\frac{\rho}{2}L^3 d \equiv N'_{\dot v} = -\pi\frac{d}{L}\{1.1\frac{B}{L} - 0.041\frac{B}{d}\} \quad (21)$$

c. Nonlinear derivatives and cross-flow drag coefficient. Inoue *et al.* have also proposed the charts for them[A.2] and Kijima *et al.* have obtained the regression forms of them[A.5].

Cross-flow drag coefficient can be assumed as

$$C_D = -Y'_{vv} \quad (22)$$

d. Rudder coefficients. If data of rudder is available, so-called MMG's model[A.6] procedure to estimate normal rudder force coefficient, effective rudder inflow velocity and angle is usable. In this paper, rudder force coefficient Y'_δ and rudder moment coefficient N'_δ were estimated to coincide with tactical diameter at rudder angle of 35 deg D_{T35}.

e. Manoeuvrability indices. Coefficients of Nomoto's 2nd order model[10], which can be written as follows, can be estimated using the above derivatives.

$$T'_1T'_2\ddot{r} + (T'_1 + T'_2)\dot{r} + r = K'\delta + K'T'_3\dot{\delta} \quad (23)$$

where

$$T'_1T'_2 = (m' + m'_y)(I'_z + J'_z)/D \quad (24)$$
$$T'_1 + T'_2 = \{-(m' + m'_y)N'_r - Y'_v(I'_z + J'_z)\}/D \quad (25)$$
$$K' = (N'_vY'_\delta - N'_\delta Y'_v)/D \quad (26)$$
$$T'_3 = (m' + m'_y)N'_\delta/(N'_vY'_\delta - N'_\delta Y'_v) \quad (27)$$

where
D : stability criterion

$$D = Y'_vN'_r + N'_v(m' + m'_x - Y'_r) \quad (28)$$

If the ship is stable ($D > 0$) and not so big, it can be more simplified into so-called Nomoto's K–T model[10] as

$$T'\dot{r} + r = K'\delta \quad (29)$$
$$T' = T'_1 + T'_2 - T'_3 \quad (30)$$

Dynamics of a steering gear is well modeled by the following equations.

$$T_E\dot{\delta} + \delta = \delta^* \quad (31)$$
$$\dot{\delta} \leq \dot{\delta}_{\max} \quad (32)$$

where
T_E : time constant of a steering gear (sec)
$\dot{\delta}_{\max}$: maximum rudder speed (deg/sec)
δ^* : rudder command

In this study, T_E is set 2.5 sec. and $\dot{\delta}_{\max}$ is set 3.5 deg/sec.

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