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Application of Linear System Analysis to Hydrodynamic Problems of a Maneuvering Ship

Masami HAMAMOTO and Kazuhiko HASEGAWA
(Department of Naval Architecture)

Abstract

The concept of linear system analysis is applied to the unsteady motion of a maneuvering ship for obtaining the hydrodynamic forces acting on her. The theoretical and practical aspects of transient motion technique are presented and discussed in detail.

1. Introduction

We shall consider the application of linear system analysis to the unsteady motion of a maneuvering ship for obtaining the hydrodynamic forces acting on her.

The frequency response characteristics of a linear system may be measured in two fundamental ways, that is to say, using sinusoidal or transient excitations. The first technique is commonly employed in the planar motion mechanism test for a ship model.[1] The latter technique, applying a known transient motion to the model and observing the hydrodynamic forces as the response, is the subject of this paper.

A transient motion contains energy which, in general, is distributed over a range of frequency. Thus a single test with a transient motion can yield information about the response of the hydrodynamic forces at any frequency of interest.

In the following sections, the theoretical and practical aspects of transient motion technique are presented. First, a mathematical model of the velocity potential for transient motion of a ship is outlined by using the integro-differential equations of the Volterra's type[2] which govern the response to an unsteady motion. Then, the equations of motion are derived, separating the effects of damping, added mass and hydrodynamic memory. Next, the captive model tests are carried out based on the transient motion technique. Finally, the experimental results are verified to satisfy the Kramers-Kronig relation between damping and added mass coefficients.

2. Equations of Motion

The equations of motion for a maneuvering ship with sway and yaw motions in smooth water may be, in general, described in the form;

$$\left.\begin{aligned} m(\dot{u}-vr) &= \iint_S p n_x \, dS + X, \\ m(\dot{v}+ur) &= \iint_S p n_y \, ds + Y, \\ I_z \dot{r} &= \iint_S p(x n_y - y n_x) \, dS + N, \end{aligned}\right\} \tag{1}$$

where m and I_z are mass and moment of inertia of a ship, and u, v and r are the linear velocities of x, y axis and the angular velocity about z axis in the co-ordinate system as shown in Fig. 1. In the right-hand side of equations, p is pressure acting on the wetted surface S of a ship hull, and n_x, n_y are x, y component of unit vector respectively. X, Y and N are external forces and moment acting on her.

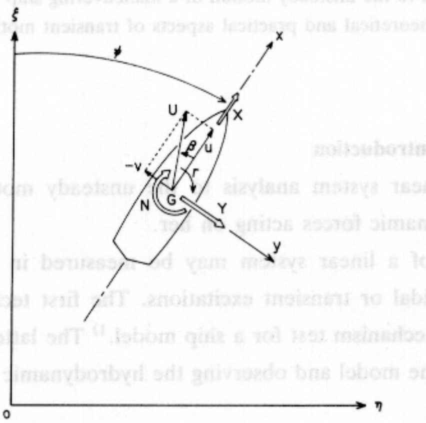

Fig. 1. Co-ordinate system and symbols.

The main problem here is the evaluation of the pressure integrals on the right-hand side of Eq. (1).

The hydrodynamic pressure acting on a maneuvering ship hull is partly due to some circulation around the ship. If so, this circulation around the body generates a vortex wake because the circulation along a closed fluid circuit is null. It seems to be a scientific approach to investigate the transport mechanism of vorticity from the boundary layer into the wake. Unfortunately, such a theoretical approach is very difficult up to the present. That is why we have chosen here another experimental approach based on the linear system analysis.

Now, the dynamic pressure $p\ (x,\ y,\ z;\ t)$ due to the ship motions in Eq. (1) is given by the Bernoulli's equation:

$$p = -\rho\phi_t + \rho(u - yr)\phi_x + \rho(v + xr)\phi_y - \frac{1}{2}\rho(\nabla\phi)^2, \tag{2}$$

where ρ is density of fluid, and ϕ and ϕ_t are velocity potential and its first-time derivative with respect to t.

Thus, it is necessary for us to describe the velocity potential for an unsteady motion of a body. The essential tool for this problem is a time-dependent Green's function $G\ (x,\ y,\ z;\ \xi,\ \eta,$

ζ; t) which corresponds to the impulse response function of a hydrodynamic impulse acting on the surface of a body moving through a fluid unbounded at infinity.

This problem has been attacked by Cummins (1962)[3] and later Wehansen (1965)[2] has provided us a physical foundation in the analytical method originally introduced by Volterra (1934).[4] The Volterra's method for the problem here starts out by applying the Green's theorem to the function ϕ_t $(x, y, z; \tau)$ and G $(x, y, z; \xi, \eta, \zeta; t\text{-}\tau)$ in the region bounded by the body surface S, the vortex wake and a large sphere of infinite radius.

Following this method, we obtain the velocity potential for an unsteady motion in the form;

$$\phi(x, y, z; t)=\frac{1}{4\pi}\int^{t}\left[\iint_{S} G(x, y, z; \xi, \eta, \zeta; t-\tau)\frac{\partial}{\partial n}\phi_t(\xi, \eta, \zeta; \tau)\, dS\right] d\tau, \tag{3}$$

where n is normal vector to be pointing out of the fluid and ξ, η, ζ and τ are integral variables. And also, we can rewrite Eq. (3) as follows:

$$\begin{aligned}\phi(x, y, z; t)=&\frac{1}{4\pi}\iint_{S}\phi_n(\xi, \eta, \zeta; t)\, G(x, y, z; \xi, \eta, \zeta; 0)\, dS\\&+\frac{1}{4\pi}\int_0^{t}\left[\iint_{S}\phi_n(\xi, \eta, \zeta; \tau)\, G_t(x, y, z; \xi, \eta, \zeta; t-\tau)\, dS\right] d\tau,\end{aligned} \tag{4}$$

where ϕ_n and G_t are first-time derivatives with respect to n and t.

The boundary conditions satisfied by ϕ $(x, y, z; t)$ are

$$\left.\begin{aligned}&\phi(x, y, z; 0)=\phi_t(x, y, z; 0)=0,\\&\phi_n(x, y, z; t)|_{\text{on } S}=un_x+vn_y+r(xn_y-yn_x), \text{ for } t>0.\end{aligned}\right\} \tag{5}$$

So that, the velocity potential ϕ $(x, y, z; t)$ can be decomposed as follows:

$$\begin{aligned}\phi(x, y, z; t)=&u\phi_1+v\phi_2+r\phi_3+\int_0^{t}u(\tau)\phi_{1t}(t-\tau)\, d\tau\\&+\int_0^{t}v(\tau)\phi_{2t}(t-\tau)\, d\tau+\int_0^{t}r(\tau)\phi_{3t}(t-\tau)\, d\tau,\end{aligned} \tag{6}$$

where the new unknown functions ϕ_k and ϕ_{kt} satisfy:

$$\left.\begin{aligned}&\frac{\partial\phi_k}{\partial n}|_{\text{on } S}=n_k,\\&\qquad\qquad\qquad(\text{for } k=1, 2, 3)\\&\frac{\partial\phi_{kt}}{\partial n}|_{\text{on } S}=0,\end{aligned}\right. \tag{7}$$

where $n_1 = n_x$, $n_2 = n_y$ and $n_3 = (xn_y - yn_x)$.

Here, assuming the perturbation velocity u, v and r from a uniform forward velocity U, we can linearize the equations of motion as follows:

$$\left.\begin{aligned}&m\dot{u}=-m_x(\infty)\dot{u}-X_u(\infty)u-\int_0^{t}u(\tau)X_u(t-\tau)\, d\tau+X,\\&m(\dot{v}+ur)=-m_y(\infty)\dot{v}-Y_v(\infty)v-\int_0^{t}v(\tau)Y_v(t-\tau)\, d\tau\\&\qquad-Y_{\dot{r}}(\infty)\dot{r}-Y_r(\infty)r-\int_0^{t}((\tau)Y_r(t-\tau)\, d\tau+Y,\end{aligned}\right\} \tag{8}$$

$$
\left.\begin{aligned}
I_z \dot{r} = & -J_z(\infty)\dot{r} - N_r(\infty)r - \int_0^t r(\tau)N_r(t-\tau)\,d\tau \\
& -N_{\dot{v}}(\infty)\dot{v} - N_v(\infty)v - \int_0^t v(\tau)N_v(t-\tau)\,d\tau + N,
\end{aligned}\right|
$$

where, being consistent with the small motion assumption, the non-linear terms on the left-hand side of Eq. (1) have been dropped.

3. Transient Motion Technique

We have made up a transient motion mechanism, as shown in Fig. 2, which gives a transient motion distributing into the frequency range of interest to a ship model. This mechanism can force surge, sway and combined motion of sway and yaw.

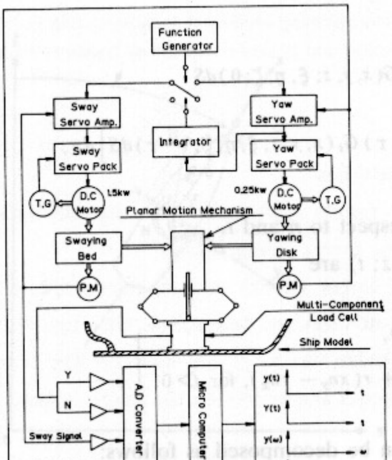

Fig. 2. Planar motion mechanism for transient maneuvering tests.

The problem now consists of finding a method for evaluating the damping and added mass coefficients as a function of the frequency. For this we shall need the Fourier transform of equations of motion.

Fourier transform of Eq. (8) yields

$$
\left.\begin{aligned}
&[X_u(\omega) + i\omega(m + m_x(\omega))][u_c(\omega) - iu_s(\omega)] = X_c(\omega) - iX_s(\omega), \\
&[Y_v(\omega) + i\omega(m + m_y(\omega))][v_c(\omega) - iv_s(\omega)] \\
&\qquad + [Y_r(\omega) + i\omega Y_{\dot{r}}(\omega)][r_c(\omega) - ir_s(\omega)] = Y_c(\omega) - iY_s(\omega), \\
&[N_v(\omega) + i\omega N_{\dot{v}}(\omega)][v_c(\omega) - iv_s(\omega)] \\
&\qquad + [N_r(\omega) + i\omega(I_z + J_z(\omega))][r_c(\omega) - ir_s(\omega)] = N_c(\omega) - iN_s(\omega),
\end{aligned}\right\} \quad (9)
$$

where subscripts c and s denote cosine and sine components of Fourier transform with respect to ω respectively. From the real and imaginary part of Eq. (9), we can derive the damping and added mass coefficients for each motion as follows:

(1) Transient surge tests

$$\left.\begin{aligned} X_u(\omega) &= \frac{X_c(\omega)u_c(\omega)+X_s(\omega)u_s(\omega)}{u_c^2(\omega)+u_s^2(\omega)}, \\ m+m_x(\omega) &= \frac{1}{\omega}\,\frac{X_c(\omega)u_s(\omega)-X_s(\omega)u_c(\omega)}{u_c^2(\omega)+u_s^2(\omega)}, \end{aligned}\right\} \qquad (10)$$

(2) Transient sway tests

$$\left.\begin{aligned} Y_v(\omega) &= \frac{Y_c(\omega)v_c(\omega)+Y_s(\omega)v_s(\omega)}{v_c^2(\omega)+v_s^2(\omega)}, \\ m+m_y(\omega) &= \frac{1}{\omega}\,\frac{Y_c(\omega)v_s(\omega)-Y_s(\omega)v_c(\omega)}{v_c^2(\omega)+v_s^2(\omega)}, \\ N_v(\omega) &= \frac{N_c(\omega)v_c(\omega)+N_s(\omega)v_s(\omega)}{v_c^2(\omega)+v_s^2(\omega)}, \\ N_{\dot v}(\omega) &= \frac{1}{\omega}\,\frac{N_c(\omega)v_s(\omega)-N_s(\omega)v_c(\omega)}{v_c^2(\omega)+v_s^2(\omega)} \end{aligned}\right\} \qquad (11)$$

(3) Transient combined motion tests

$$\left.\begin{aligned} Y_r(\omega) &= \frac{Y_c^*(\omega)r_c(\omega)+Y_s^*(\omega)r_s(\omega)}{r_c^2(\omega)+r_s^2(\omega)}, \\ Y_{\dot r}(\omega) &= \frac{1}{\omega}\,\frac{Y_c^*(\omega)r_s(\omega)-Y_s^*(\omega)r_c(\omega)}{r_c^2(\omega)+r_s^2(\omega)}, \\ N_r(\omega) &= \frac{N_c^*(\omega)r_c(\omega)+N_s^*(\omega)r_s(\omega)}{r_c^2(\omega)+r_s^2(\omega)} \\ I_z+J_z(\omega) &= \frac{1}{\omega}\,\frac{N_c^*(\omega)r_s(\omega)-N_s^*(\omega)r_c(\omega)}{r_c^2(\omega)+r_s^2(\omega)}, \end{aligned}\right\} \qquad (12)$$

where,

$$\left.\begin{aligned} Y_c^*(\omega) &= Y_c(\omega)-Y_v(\omega)v_c(\omega)-\omega[m+m_y(\omega)]v_s(\omega), \\ Y_s^*(\omega) &= Y_s(\omega)-Y_u(\omega)v_s(\omega)+\omega[m+m_y(\omega)]v_c(\omega), \\ N_c^*(\omega) &= N_c(\omega)-N_v(\omega)v_c(\omega)-\omega N_{\dot v}(\omega)v_s(\omega), \\ N_s^*(\omega) &= N_s(\omega)-N_s(\omega)-N_v(\omega)v_s(\omega)+\omega N_{\dot v}(\omega)v_c(\omega). \end{aligned}\right\} \qquad (13)$$

Now, model tests have been conducted, following the analysis mentioned above. A model of a container ship, whose dimensions and profile are shown in Table 1 and Fig. 3 respectively, was used for experiments. The model ship is towed at the velocity of 1.11m/s, or Froude number is 0.224.

The time history of a transient sway test is shown in Fig. 4. The results obtained from the transient sway tests of several drift angles and from the conventional PMM tests are shown in Fig. 5. Similarly, in Fig. 6 derivatives obtained from transient combined motion tests are shown. For all cases the results are of good coincidence with those by the conventional PMM tests marked by circles in the figures.

Table 1. Principal particulars of the ship and the model.

Items		Ship	Model
Length	(m)	115.0	2.5
Breadth	(m)	19.0	0.413
Draft	(m)	6.4	0.139
Displaced volume	(m)	9859.0	0.101
Wetted surface area	(m)	2845.0	1.345
Block coefficient		0.705	0.705
Midship section coef.		0.970	0.970
Breadth/Draft ratio		2.97	2.97
Length/Breadth ratio		6.05	6.05
Model Scale		—	1/46

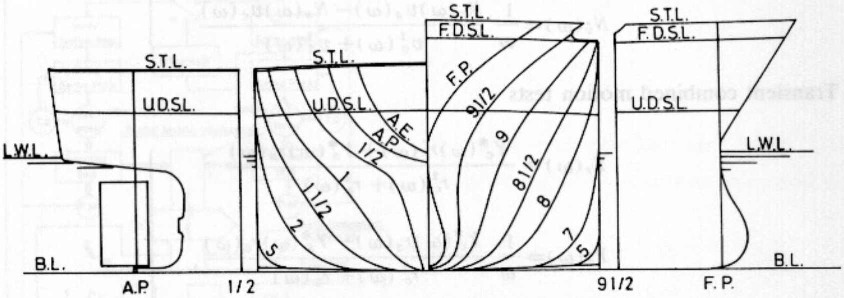

Fig. 3. Body plan, and bow and stern profile of the ship model.

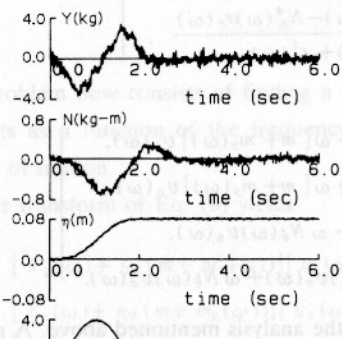

Fig. 4. Time history of a transient sway test.

The experimental results are verified by the Kramers-Kronig relation. In a causal system, there is a so-called Kramers-Kronig relation between added mass and damping coefficients. One of them can be determined from the other by using the Hilbert transform described by Eq. (14).

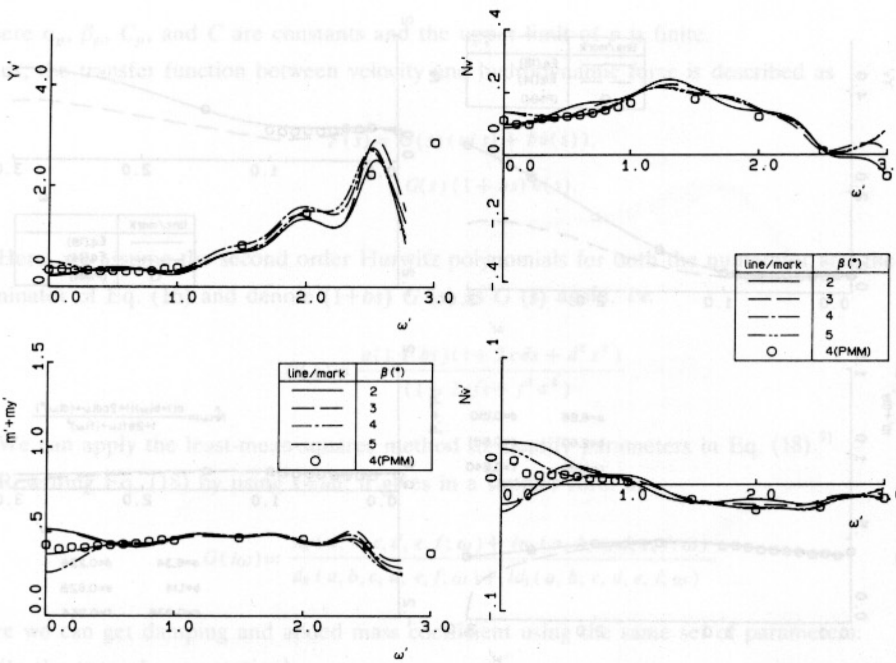

Fig. 5. Results of transient sway tests.

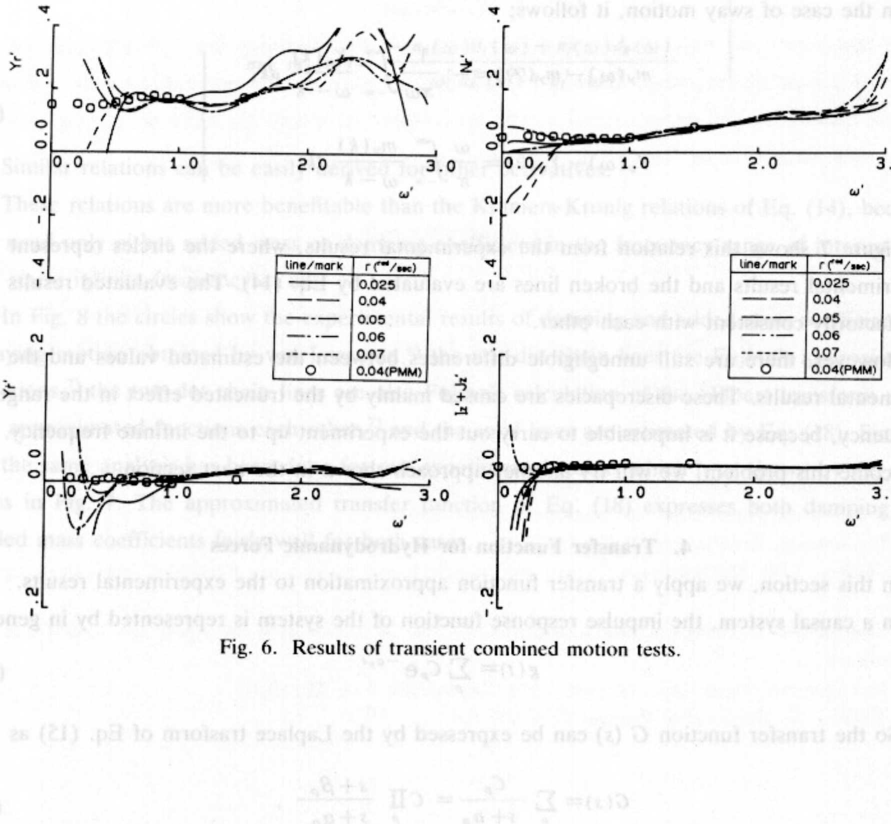

Fig. 6. Results of transient combined motion tests.

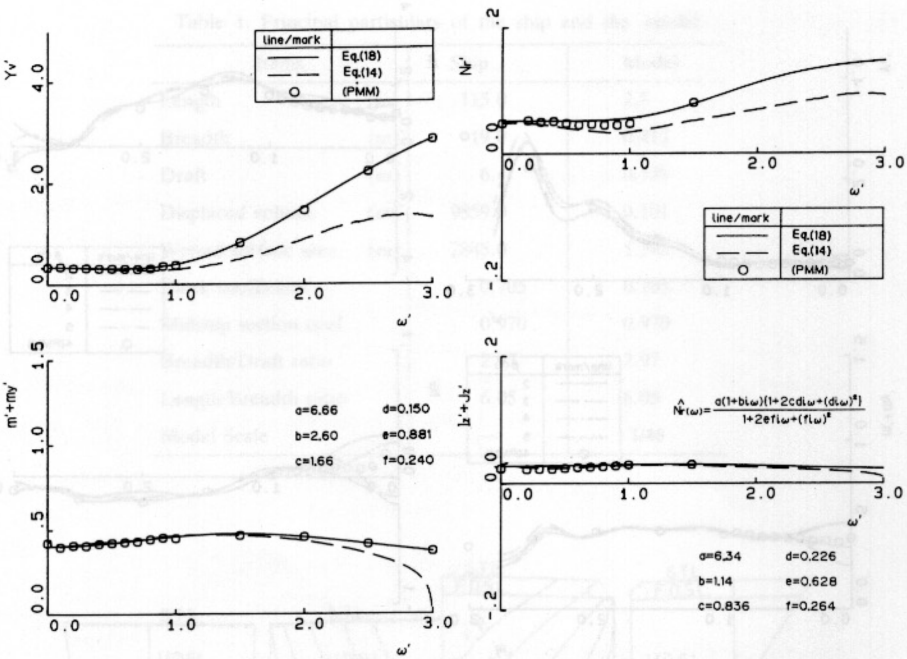

Fig. 7. Kramers-Kronig relation.

In the case of sway motion, it follows;

$$\left.\begin{aligned} m_y(\omega)-m_y(\infty) &= -\frac{1}{\pi\omega}\int_{-\infty}^{\infty}\frac{Y_v(k)}{\omega-k}\,dk, \\ Y_v(\omega)-Y_v(0) &= \frac{\omega}{\pi}\int_{-\infty}^{\infty}\frac{m_y(k)}{\omega-k}\,dk. \end{aligned}\right\} \tag{14}$$

Figure 7 shows this relation from the experimental results, where the circles represent the experimental results and the broken lines are evaluated by Eq. (14). The evaluated results are satisfactorily consistent with each other.

However, there are still unnegligible differences between the estimated values and the experimental results. These discrepacies are caused mainly by the truncated effect in the range of frequency, because it is impossible to carry out the experiment up to the infinite frequency. To overcome this problem, we will try another approach shown in the next section.

4. Transfer Function for Hydrodynamic Forces

In this section, we apply a transfer function approximation to the experimental results.

In a causal system, the impulse response function of the system is represented by in general

$$g(t)=\sum_{p} C_p e^{-\alpha_p t}. \tag{15}$$

So the transfer function $G(s)$ can be expressed by the Laplace trasform of Eq. (15) as

$$G(s)=\sum_{p}\frac{C_p}{s+\alpha_p}=C\prod_{p}\frac{s+\beta_p}{s+\alpha_p}\ , \tag{16}$$

where α_p, β_p, C_p, and C are constants and the upper limit of p is finite.
Thus, the transfer function between velocity and hydrodynamic force is described as

$$\begin{aligned} F(s) &= G(s)\,(u(s)+b\dot{u}(s)), \\ & G(s)\,(1+bs)\,u(s). \end{aligned} \tag{17}$$

Here, we assume the second order Hurwitz polynomials for both the numerator and the denominator of Eq. (16) and denote $(1+bs)\ G\ (s)$ as $G\ (s)$ again, *i.e.*

$$G(s)=\frac{a(1+bs)(1+2cds+d^2s^2)}{(1+2efs+f^2s^2)}. \tag{18}$$

We can apply the least-mean-squares method to identify parameters in Eq. (18).[5]
Rewriting Eq. (18) by using $s=i\omega$, it gives in a simpler form as

$$G(i\omega)=\frac{n_R(a,b,c,d,e,f;\omega)+in_I(a,b,c,d,e,f;\omega)}{d_R(a,b,c,d,e,f;\omega)+id_I(a,b,c,d,e,f;\omega)}. \tag{19}$$

Here we can get damping and added mass coefficient using the same set of parameters:
(In the case of sway motion)

$$\left.\begin{aligned} Y_v(\omega) &= \frac{n_R(\omega)d_R(\omega)+n_I(\omega)d_I(\omega)}{d_R^2(\omega)+d_I^2(\omega)}, \\ m+m_y(\omega) &= \frac{-n_R(\omega)d_I(\omega)+n_I(\omega)d_R(\omega)}{\omega[d_R^2(\omega)+d_I^2(\omega)]}. \end{aligned}\right\} \tag{20}$$

Similar relations can be easily derived for other derivatives.

These relations are more benefitable than the Kramers-Kronig relations of Eq. (14), because we need only either added mass or damping coefficient in the frequency range of interest, but not up to infinite frequency at all.

In Fig. 8 the circles show the experimental results of damping and added mass coefficients of swaying motion obtained by van Leeuwen,[6] the one-dot chain lines are Fujino's approximated functions,[7] the two-dot chain lines are also Fujino's calculation of the Hilbert transform using the approximated functions each other,[7] and the solid lines are estimated by Eq. (18). Similarly, the same analysis has been done for our experimental results which is as shown by the solid lines in Fig. 7. The approximated transfer function of Eq. (18) expresses both damping and added mass coefficients fairly well for both cases.

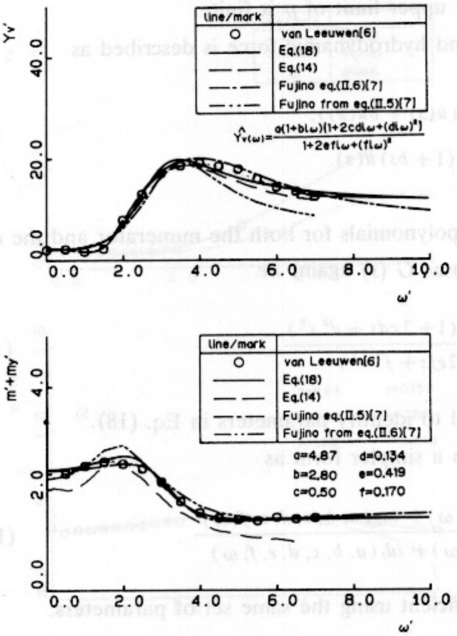

Fig. 8. Transfer function approximation of hydrodynamic coefficients.

5. Conclusion

In this paper, we have presented a new technique for obtaining the hydrodynamic coefficients of a maneuvering ship. The captive model tests were carried out based on the transient motion technique. The experimental results are verified to satisfy the Kramers-Kronig relation and a new method using the transfer function was proposed in this paper.

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