J. Kansai Soc. N. A., Japan, No. 178, September 1980

On a Performance Criterion of Autopilot Navigation*

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A performance criterion of autopilot navigation is a long-discussed topic but remains still fresh. It is necessary to carry out model experiments to evaluate the criterion and obtain the weighting factors of the criterion. But because accurate measurement of longitudinal hydrodynamic force was difficult, most researches ended in qualitative approach.

In the present paper, we derive the performance criterion from the concept of energy increase due to yawing. The criterion is found to consist of two parts ; elongation of sailing distance and thrust increase due to steering and yawing motions. The former is proportional to rms value of course deviation and the latter, depending upon ship dynamics, is given by rms values of rudder angle and rate of turn. Each weighting factor to rms values is obtained by model experiments. Some comments on the estimation and the scaling-up methods of derivatives are given.

1. Introduction

Since the energy crisis in 1973, economic navigation of ships is further demanded. Power increase due to yawing in open sea induced by steering becomes unnegligible. This power increase depends both upon ship's course stability and autopilot performance : the less stability and the poorer autopilot result in the greater power increase.

The recent trend in ship design brought out fuller and beamier ships with poorer course stability. Naval architects are anxious about if such poor stability introduces an appreciable loss of propulsive power. We should have some means of evaluating power loss induced by yawing at sea, as a function of ship's course stability. At the same time, such evaluation will certainly be useful to refine autopilot performance.

We have already some criteria along this line. Nomoto and Motoyama (1966)[1] evaluated the power increase due to the self-sustaining oscillation which occurs unavoidably by "weather adjust" mechanism in autopilots. On the other hand, Koyama (1967)[2] derived a criterion from resistance increase in a wider sense. He considered the centrifugal force due to yawing is negligible if the system stability is adequate. Similar approach was made by Norrbin (1972)[3], mainly focusing the power loss in steering at the mile-post speed trial. Kose and Sasaki (1976)[4] proposed another criterion combining above criteria, which is the base of the present formula.

In this paper, a performance criterion according to the energy increase due to autopilot steering in open sea is discussed. The weighing factors, which are essential in evaluating the yawing power loss at sea, are given by means of captive model tank tests.

2. Energy saving index as a cost function

2. 1 Why do we employ Energy saving index ?

Schilling (1976)[5] regarded the loss of distance travelled as a cost function and Walters (1977)[6] used the time loss in covering a certain distance at a given rate of fuel consumption. Recently Kanamaru and Sato (1979)[7] provided the quite similar cost function as the present formula derived from the energy increase.

Speed decrease, fuel consumption are also worth considering as a cost function of autopilot navigation. Although the subjects of cost functions are different, the final form of the function is settled into three types, which correspond to equations derived by Nomoto[1], Koyama[2], and

*Read at the Spring Meeting of the Kansai Society of Naval Architects, Japan, May 23, 1980.

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Kose[4] respectively. They are

$$J=\lambda_2\overline{\delta^2}+\lambda_3\overline{r'^2} \quad \cdots\cdots\cdots(1)$$

$$J=\lambda_1\overline{\psi^2}+\lambda_2\overline{\delta^2} \quad \cdots\cdots\cdots(2)$$

$$J=\lambda_1\overline{\psi^2}+\lambda_2\overline{\delta^2}+\lambda_3\overline{r'^2} \quad \cdots\cdots\cdots(3)$$

where λ_1, λ_2, λ_3 : weighting factors

$\overline{\psi^2}$: rms value of course deviation (rad^2)

$\overline{\delta^2}$: rms value of rudder angle (rad^2)

$\overline{r'^2}$: rms value of rate of turn (rad^2)

In the case of speed decrease, for example, the cost function depends upon not only the ship dynamics but also the kind of the main engine installed. From the view point of voyage expenses, fuel consumption seems to be a suitable parameter, but it also varies with the kind of the engine and the speed governor. The situation is more or less similar in other physical terms. It is not agreeable to adopt these parameters as the cost function, which had better be affected only by the ship dynamics.

Besides, any motions that cannot be reduced by any adjustment of autopilots should not be taken account of. They are, for example, heaving and pitching motions as well as yawing motion directly caused by wave. Speed drop caused by these motions should be also out of consideration.

Through these considerations, the energy necessary to convey a ship from one port to the other is selected for the most suitable index to evaluate the autopilot navigation in open sea.

2. 2 Energy increase in open sea

The total energy E required for travelling from one port to the other is defined as

$$E=\int_0^{\tau} T\cdot V dt=\overline{T}\cdot S \quad \cdots\cdots\cdots(4)$$

where T : propeller thrust force

V : ship speed

τ : sailing time

S : sailing distance

— over a symbol denotes mean value.

If the ship sails straightly without any steering in calm sea, using the subscript 0, the total energy is

$$E_0=\int_0^{\tau_0} T_0\cdot V_0\, dt=T_0\cdot S_0 \quad \cdots\cdots\cdots(5)$$

In the actual voyage, ship speed fluctuates by yawing. Expressing them by the increment from the value of straight running in calm sea, the total energy in open sea is given as

$$\begin{aligned}E&=\int_0^{\tau_0+\Delta\tau}(T_0+\Delta T)(V_0+\Delta V)\, dt\\&=(T_0+\overline{\Delta T})\,(S_0+\Delta S)\\&\equiv E_0+\Delta E\end{aligned} \quad \cdots\cdots\cdots(6)$$

Then we can define the energy increase due to yawing, neglecting the second-order terms :

$$J=\frac{E-E_0}{E_0}=\frac{\Delta E}{E_0}\doteqdot\frac{T_0\Delta S+\overline{\Delta T}S_0}{T_0 S_0}=\frac{\Delta S}{S_0}+\frac{\overline{\Delta T}}{T_0} \quad \cdots\cdots\cdots(7)$$

We now find the performance criterion is divided into two parts ; elongation of sailing distance and thrust increase.

2. 3 Elongation of sailing distance

Koyama (1967)[2] first treated elongation of sailing distance as resistance increase in a wider sense. Considering the sinusoidal motion, he gave elongation of sailing distance by line integral along the path :

$$\frac{\Delta S}{S_0}=\frac{1}{2}\overline{\psi^2} \quad \cdots\cdots\cdots(8)$$

We can derive the similar expression from the integration of ship speed.

Regarding actual distance sailed during yawing is

$$S=S_0+\Delta S=\int_0^{\tau} V dt=\overline{V}\cdot\tau \quad \cdots\cdots\cdots(9)$$

and straight distance S_0 is

$$\begin{aligned}S_0&=\int_0^{\tau} V\cdot\cos\psi \mathrm{d}t=\overline{V}\int_0^{\tau}\cos\psi \mathrm{d}t\\&\doteqdot\overline{V}\tau(1-\frac{1}{2}\overline{\psi^2})\end{aligned} \quad \cdots\cdots\cdots(10)$$

then, we get elongation of sailing distance as

$$\frac{\Delta S}{S_0}=\frac{\frac{1}{2}\overline{\psi^2}}{1-\frac{1}{2}\overline{\psi^2}}\doteqdot\frac{1}{2}\overline{\psi^2}=\frac{\Delta S}{S} \quad \cdots\cdots\cdots(11)$$

Error of approximation in eq. (11) is about proportional to yawing amplitude. Error at 20° yawing is about 3% and error at ordinary yawing is small enough to ignore.

2. 4 Thrust increase due to autopilot steering

We describe the equation of longitudinal ship motion on the body-fixed coordinate system (Fig. 1). According to the recommendation of Group-MMG[8], we get

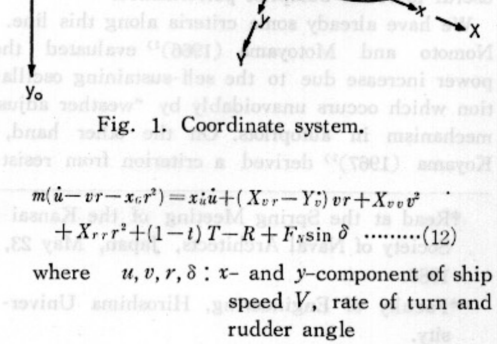

Fig. 1. Coordinate system.

$$m(\dot{u}-vr-x_G r^2)=x_{\dot{u}}\dot{u}+(X_{vr}-Y_{\dot{v}})vr+X_{vv}v^2 +X_{rr}r^2+(1-t)T-R+F_N\sin\delta \quad \cdots\cdots\cdots(12)$$

where u, v, r, δ : x– and y–component of ship speed V, rate of turn and rudder angle

x_G : x-coordinate of the centre of gravity

m : mass

$-X_{\dot{v}}, -Y_{\dot{v}}$: added mass in x- and y-direction

X_{vr}, X_{vv}, X_{rr} : coefficients of resistance increment due to ship motions (v, r)

t : thrust deduction factor

T : propeller thrust force

R : hull resistance in straight running, including rudder resistance with helm amidship

F_N : rudder normal force

For simplicity, the forward-speed equation will be referred to the centre of gravity of the ship, so that $x_G=0$.

Rudder normal force F_N is given as

$$F_N=\frac{\rho}{2}A_R U_R^2 f_a(\Lambda)\,\alpha_R \qquad \cdots\cdots(13)$$

where A_R : rudder area

Λ : aspect ratio of rudder

$f_a(\Lambda)$: slope of the open-water characteristics of rudder normal force

α_R, U_R : effective attack angle of flow into rudder and inflow velocity (u_R : x-component of U_R)

As an expression of $f_a(\Lambda)$, Fujii's formula[9], based on semi-theoretical background, is popular in practical use :

$$f_a(\Lambda)=\frac{6.13\Lambda}{\Lambda+2.25} \qquad \cdots\cdots(14)$$

when the ship motion is not so large, we can treat α_R as $-\delta$ and

$$U_R \doteqdot u_R=(1-w)\,V\,\varepsilon\sqrt{1+\kappa\frac{8}{\pi}\frac{K_T}{J^2}} \qquad \cdots\cdots(15)$$

where w : effective wake fraction

ε, κ : empirical constants

K_T : propeller thrust coefficient

J : advance ratio

Therefore we define the resistance due to steering as

$$F_N\sin\delta=-R_{\delta\delta}\delta^2=-R_{\delta\delta 0}\left(\frac{V}{V_0}\right)^2\delta^2 \qquad \cdots\cdots(16)$$

where $R_{\delta\delta}=\frac{\rho}{2}A_R(1-w)^2V^2\varepsilon^2\left(1+\kappa\frac{8K_T}{\pi J^2}\right)\frac{6.13\Lambda}{\Lambda+2.25}$

$R_{\delta\delta 0}=\frac{\rho}{2}A_R(1-w)^2V_0^2\varepsilon^2\left(1+\kappa\frac{8K_T}{\pi J^2}\right)\frac{6.13\Lambda}{\Lambda+2.25}$

Hull resistance is satisfactorily expressed as

$$R=-X_{uu}u^2 \qquad \cdots\cdots(17)$$

In straight running, $T_0(1-t)=R_0(V_0)$ and we can assume $(1-t)$ is constant even during yawing. As the average variation of u is zero and the increment of hull resistance is $-2X_{uu}V_0\,\overline{\Delta V}$, thrust increase due to yawing is

$$\frac{\Delta T}{T_0}=-\frac{\overline{(m+X_{vr}-Y_{\dot{v}})vr+X_{vv}v^2+X_{rr}r^2}}{R_0(V_0)}+\frac{\overline{R_{\delta\delta}\delta^2}}{R_0(V_0)}+2\frac{\overline{\Delta V}}{V_0} \qquad \cdots\cdots(18)$$

In the actual sea state, ship motions under autopilot steering distribute in a wide frequency band. Even in the self-sustaining oscillation such as shown in Fig. 2, we cannot regard it as a

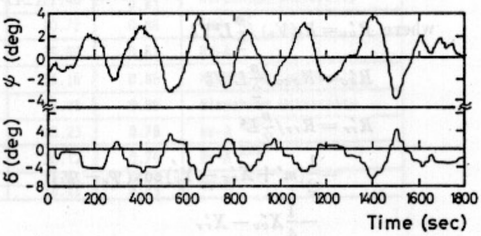

Fig. 2. Example of self-sustaining oscillation observed on an ore carrier[22].

simple harmonic motion. Theory of Fourior series expansion, however, provides us a simplified treatment. If the system nonlinearity is small enough, we can superpose input-output relation. As in the present case the motion is not so large, we treat only single sine wave as the motion hereafter.

Let the rudder motion be

$$\delta=\delta_0\sin\omega t \qquad \cdots\cdots(19)$$

where δ_0 : amplitude of rudder motion (rad)

ω : frequency of rudder motion (rad/sec)

t : time (sec)

Then the yawing and swaying motions caused by the rudder motion are

$$\left.\begin{aligned} r&=r_0\sin(\omega t+\varphi_r)\\ v&=v_0\sin(\omega t+\varphi_v)\end{aligned}\right\} \qquad \cdots\cdots(20)$$

where r_0, v_0 : amplitude of yawing (rad/sec) and swaying (m/sec) motions

φ_r, φ_v : phase lag of yawing and swaying motions to rudder motion (rad)

Applying eqs. (19) and (20) into eq. (18), we get

$$\frac{\Delta T}{T_0}=-\frac{(m+X_{vr}-Y_{\dot{v}})}{2R_0(V_0)}v_0r_0\cos(\varphi_v-\varphi_r)-\frac{X_{vv}}{2R_0(V_0)}v_0^2-\frac{X_{rr}}{2R_0(V_0)}r_0^2+\frac{R_{\delta\delta}}{2R_0(V_0)}\delta_0^2+\left(\frac{\overline{V}}{V_0}\right)^2-1 \qquad \cdots\cdots(21)$$

It is reasonable to substitute swaying motion for yawing motion as

$$v_0\doteqdot-\frac{L}{2}r_0 \qquad \cdots\cdots(22)$$

and we define

$$R_{rr}=\frac{L}{2}(m+X_{vr}-Y_{\dot{v}})\cos(\varphi_v-\varphi_r)-\frac{L^2}{4}X_{vv}-X_{rr} \qquad \cdots\cdots(23)$$

Nondimensionalizing the mass by $\frac{\rho}{2}$ L^3 etc. and using rms values, we finally get

$$\frac{\overline{\Delta T}}{T_0}=\frac{R'_{\delta\delta}}{R'_{uu}}\left(\frac{V}{V_0}\right)^2\overline{\delta^2}+\frac{R'_{rr}}{R'_{uu}}\left(\frac{V}{V_0}\right)^2\overline{r'^2}+\left(\frac{V}{V_0}\right)^2-1 \qquad \cdots\cdots(24)$$

where $R'_{uu}=R_0(V_0)/\frac{\rho}{2}L^2V_0^2$

$$R'_{\delta\delta}=R_{\delta\delta 0}/\frac{\rho}{2}L^2V_0^2$$

$$R'_{rr}=R_{rr}/\frac{\rho}{2}L^4$$

$$=\frac{1}{2}(m'+X'_{vr}-Y'_{\dot{v}})\cos(\varphi_v-\varphi_r)-\frac{1}{4}X'_{vv}-X'_{rr}$$

2. 5 Performance criterion under constraint

Together with eqs. (8) and (24), we obtain the performance criterion eq. (7). It is obvious from eq. (24) that thrust increase depends upon the speed variation as well as rms values of r' and δ. If we release from any limitation to speed drop, there arises a queer result: the lower the ship speed is, the lower is the energy increase. Therefore we have to add a certain constraint on the criterion.

If we control the ship speed to be constant, eq. (7) will be

$$J=\frac{1}{2}\overline{\psi^2}+\frac{R'_{\delta\delta}}{R'_{uu}}\overline{\delta^2}+\frac{R'_{rr}}{R'_{uu}}\overline{r'^2} \qquad \cdots\cdots(25)$$

Another proper constraint may be to control the ship on schedule. In this case, speed increment is

$$\overline{\Delta V}=\frac{\Delta S}{\tau_0}=\frac{\Delta S}{S_0}\cdot\frac{S_0}{\tau_0}=\frac{1}{2}\overline{\psi^2}\cdot V_0 \qquad \cdots\cdots(26)$$

Substituting eq. (26) into eq. (24) and neglecting the second-order terms, the criterion will be

$$J=\frac{3}{2}\overline{\psi^2}+\frac{R'_{\delta\delta}}{R'_{uu}}\overline{\delta^2}+\frac{R'_{rr}}{R'_{uu}}\overline{r'^2} \qquad \cdots\cdots(27)$$

The difference occurs only on the weighting factor of course deviation and no term of ship speed is included. This seems very convenient, because the measurement or calculation of ship speed caused only by ship motions due to steering is troublesome. Even in the case of on-board analysis, we can employ the average ship speed $\overline{V}$ instead of approach speed V_0.

3. Measurement and estimation of longitudinal ship motions

3. 1 Model experiments

We now have the performance criterion of autopilot navigation through the discussions in the former section. But we don't know thoroughly about each derivative reffered in the criterion.

For the purpose of determining and estimating the weighting factors in the criterion, we conducted model experiments. Captive model tests using Planar Motion Mechanism (PMM) at Hiroshima University and Circular Motion Mechanism (usually the test using this facility is called CMT) at University of Tokyo were carried out. Table 1 shows particulars of tested model ships as well as obtained derivatives etc.

Table 1. Principal particulars and derivatives of tested model ships.

ITEM \ MODEL NAME	model A	model B	model C
kind of ship	Cargo Ship	Tanker	Tanker
L_{PP} (m)	4.000	3.000	4.000
B_m (m)	0.571	0.467	0.800
d_m (m)	0.229	0.181	0.253
C_B	0.700	0.810	0.827
F_n	0.200	0.163	0.137
Λ	2.17	1.40	——
$f_\alpha(\Lambda)$	3.01	2.35	1.96 *
ε	0.868	1.408	1.759
$\varkappa$	0.675	0.180	0.059
K_T **	0.209	0.251	0.241
J **	0.741	0.274	0.300
$1-w$ **	0.665	0.395	0.497
$-Y'_{\dot{v}}\times 10^3$	11.40	13.16	15.98
$X'_{vr}\times 10^3$	-1.95	-1.88	-1.92
$X'_{vv}\times 10^3$	-2.56	-2.73	——
$X'_{rr}\times 10^3$	-0.802	0.673	2.03
$R'_{uu}\times 10^3$	0.796	0.978	1.35

* : obtained from experiments

** : ship point

Usually it is not easy to measure x-direction force under ship motions. Propeller thrust balances with hull resistance in straight running, so that we have to measure small variation from the balance during small ship motions. We present the results of model experiments first. We provide next the method to estimate each weighting factor with help of former researches and results of other facilities.

3. 2 Resistance increment due to ship motions

As we can see from eq. (18), resistance increment due to ship motions (v, r) is represented by the term

$$(m+X_{vr}-Y_{\dot{v}})\,vr+X_{vv}v^2+X_{rrr}r^2 \qquad \cdots\cdots(28)$$

It is well known the axial component of the centrifugal force, corresponding to $(m-Y'_{\dot{v}})vr$ is dominant and that rest terms ore often considered to be negligible. In fact, it can be confirmed from Table 1 that major part of resistance incre-

Table 2. Principal particulars and derivatives (X_{vr}', $Y_{\dot{v}}'$) of referred model ships.

KIND OF SHIP \ ITEM	L_{pp} (m)	B_m (m)	d_m (m)	C_B	$X'_{vr} \times 10^3$	$-Y'_{\dot{v}} \times 10^3$	c_m	Data Source
RO/RO	3.000	0.580	0.132	0.530	-2.90	6.62	0.56	Hiroshima University
Car Ferry	1.500	0.241	0.061	0.531	-2.87	5.21	0.45	SRI [15]
RO/RO	3.000	0.568	0.132	0.541	-3.91	5.21	0.44	Horoshima University
Container	5.000	0.726	0.243	0.562	-3.10	7.04	0.56	NKK [17]
Cargo	6.437	0.927	0.299	0.597	-2.07	7.40	0.72	Hy-A [14]
Container	7.583	0.895	0.305	0.597	-3.89	6.60	0.41	Hy-A [12]
Cargo	4.000	0.571	0.299	0.700	-2.93 -1.95	11.40	0.77 0.83	Hiroshima University [16]*
Tanker	4.018	0.538	0.227	0.797	-1.29	10.72	0.88	TNO [13]
Tanker	7.652	1.333	0.471	0.799	-5.80	15.30	0.62	Hy-A **
Tanker	3.000	0.467	0.181	0.810	-1.88	13.16	0.86	Hiroshima University
Tanker	4.000	0.800	0.400	0.827	-1.92	15.98	0.88	Hiroshima University
Tanker	7.684	1.318	0.542	0.829	-4.41	20.23	0.78	Hy-A [12]
Tanker	7.684	1.318	0.542	0.829	-5.22	20.12	0.74	Hy-A [12]
Tanker	8.190	1.231	0.423 0.493	0.836 0.813	-2.46 -4.23	12.21 15.91	0.80 0.73	Hy-A ***

ment is the x-component of the centrifugal force. But it is true that the contribution of X_{vr} is not so small.

Added mass in y-direction $(-Y_{\dot{v}})=m_y$ can be obtained from pure swaying test using PMM. It is widely known that we can estimate this value from Motora's chart[10].

On the other hand, information as to the derivative X_{vr} is poor. Because the viscosity of flow and/or effect of free surface cause this term, it was not recognized but treated as zero (in ideal flow) or even $(-Y_{\dot{v}})$ was ignored for the long time. Utilizing the data of speed during the steady turning at full-scale trials, however, Norrbin (1971)[11] suggested for the first time, with a wide scatter, most probable values of $X_{vr}'-Y_{\dot{v}}'$ come up to be of the order of 20 to 50 per cent of $(-Y_{\dot{v}})$.

According to the development in the technique of captive model tests, some European model basins succeeded to measure this coefficient around 1970. Wagner Smitt and Chislett (1972)[12] showed the values of one container vessel and two tankers in the form of the ratio $(X_{vr}'-Y_{\dot{v}}')/(-Y_{\dot{v}}')$. The fully-investigated works made by Leeuwen and Journée (1972)[13] and Wagner Smitt and Chislett (1974)[14] also contain the data of X_{vr}'. In Japan Ogawa (1973)[15], chairman of the Group-MMG, Kose (1979)[16] and Matsumoto (1980)[17] contributed to this coefficient.

Together with these data and data presented in Table 1 the author (1980)[18] proposed the estimation chart of X_{vr}'. The result is reappeared in Table 2 and Fig. 3 with some additional plots. Although data of this coefficient are not sufficient in number and are scattering within the region enveloped by the broken line, we can observe the proportional relation between the ratio $(X_{vr}'-Y_{\dot{v}}')/(-Y_{\dot{v}}')$ and block coefficient C_B. The full line, which is not determined by LSM, is expected to provide the most possible value. If rough treatment is allowed, we may substitute the correction factor $c_m=(X_{vr}-Y_{\dot{v}})/(-Y_{\dot{v}})$ by the same value with C_B.

Increments of hull resistance due to swaying and yawing are of secondary importance, so that few data was available ten years ago. Jinnaka and others (1962)[19] proposed an expression of X_{vv} from the experience of aeroplanes. Norrbin (1971)[11] offered an approximate value of X_{vv} from some model experiments. Walters (1977)[6], on the other hand, derived the equation of X_{vv} and X_{rr} from the axial component of lift force acting on a bare hull. Unfortunately few certification was given for these expressions at these times.

Figs. 4, 5 and 6 show results of model experiments, where β denotes drift angle in degrees and v_0 and r_0 are swaying and yawing amplitude respectively. X_{rr}' is readable from the curves connecting the mark ○. X_{vv}', which is derived from the values on ordinate, is not easy to iden-

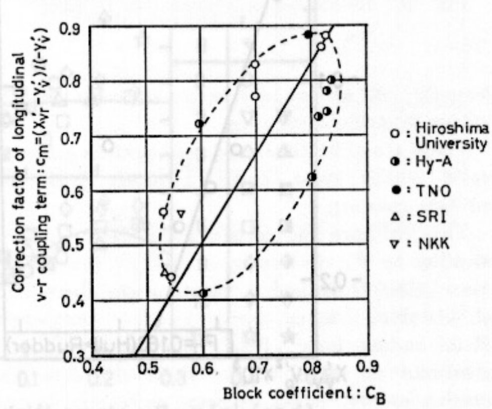

Fig. 3. Estimation chart of longitudinal v-r coupling term[18].

tify from the figures. Concerning to the expression of these terms, many experiments such as Figs. 4, 5 and 6 verify the propriety of eq. (12), though the coefficients X_{vv}' and X_{rr}' don't seem to have the uniform tendency but sometimes change even their signs. Of course, we should note that the relations are limited within some small ranges of swaying and yawing velocity and that further investigation is waited for.

The last component to be considered is the phase lag term of sway to yaw. This term enables to separate resistance components caused by ship motions due to steering and ship motions directly induced by wave. This problem was already discussed by Nomoto[1] and Koyama[2].

The phase lag of sway to yaw caused by steering is usually evaluated by steering induces T_3 and $T_{3\beta}$ and roughly shown in Fig. 7 as an example. As dominant frequency of steering and therefore of ship motions due to steering is around $\omega = 0.01$, the effect of the phase lag term is not so large but about 0.85 in the worst. Besides, it is not easy to estimate this term accurately and to evaluate the rms value of this term in general, so that we assume this term is 1 in the present analysis.

Phase lag of sway to yaw directly induced by wave, on the other hand, is usually 90 degrees, so that the vector product of sway and yaw motions is almost zero. Therefore we can only

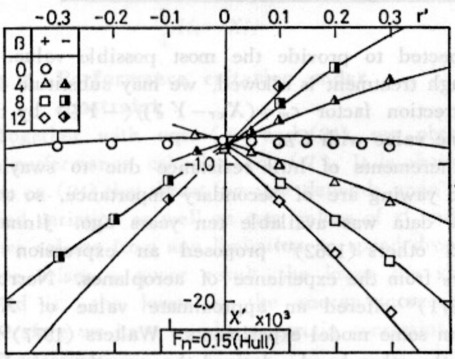

Fig. 4. Resistance increment due to ship motions measured by CMT (model A)[16].

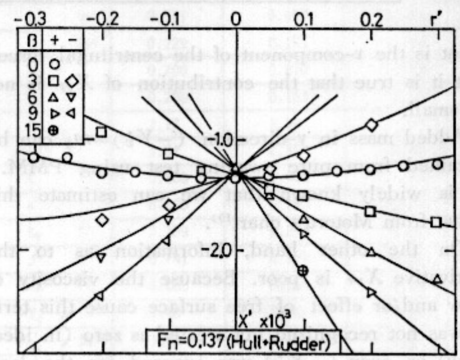

Fig. 6. Resistance increment due to ship motions measured by CMT (model C).

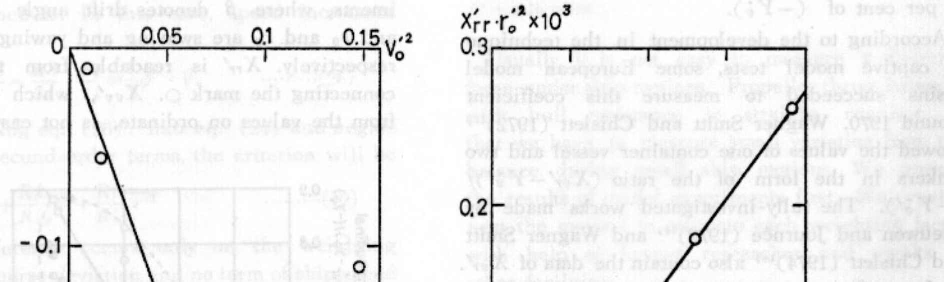

Fig. 5. Resistance increment due to ship motions measured by PMM (model B).

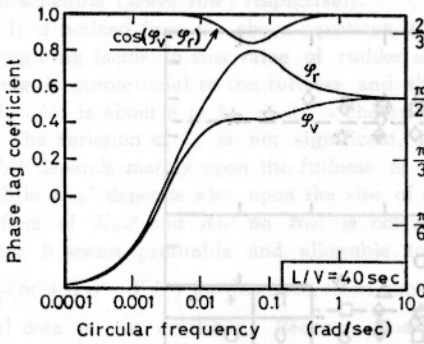

Fig. 7. Example of phase lag of sway and yaw to rudder motion.

take care of ship motions due to steering as the resistance increment caused by the centrifugal force. We should note, however, Koyama[2] pointed out that at some following seas, there are some frequency zone where the centrifugal force due to wave excitation amounts to an unnegligible value.

3. 3 Resistance increment due to steering

Another importance is laid on the resistance directly caused by steering. This component of induced drag is known to be proportional to the square of rudder angle, just the same as in the case of resistance increment due to ship motions. Hence, it is possible to obtain the derivative $R_{\delta\delta}'$ from the model experiments. But in the case of the rudder, it is easy to estimate the force acting on the rudder from the wing theory, so that there are already some empirical and/or theoretical expressions of the open-water and interaction characteristics of a rudder.

MMG's model[8], of course, contains an expression of the rudder force. The model is composed of the open-water characteristics, inflow angle and velocity of a rudder. This is expressed in eq. (13).

In the present analysis we need the longitudinal component of rudder normal force, while the MMG's model is aiming mainly at the lateral component. As the latter component is far larger than the former and the problem is restricted within small ship motions, we make some simplifications from the MMG's model. We ignore the effects of ship motions on inflow attack angle and velocity as described in 2. 4.

Besides, it is known that ship motions affect both on wake fraction and thrust deduction. Figs. 8 and 9 show an example. In the figures $J_s = V/nD$ and $\beta - x_P' r'$ (x_P' ; empirical constant) denote apparent advance ratio and equivalent drift angle at the propeller respectively. Effect of ship motions on wake fraction is generally complicated and differs among ships, though that on thrust deduction is not significant. At the present analysis,

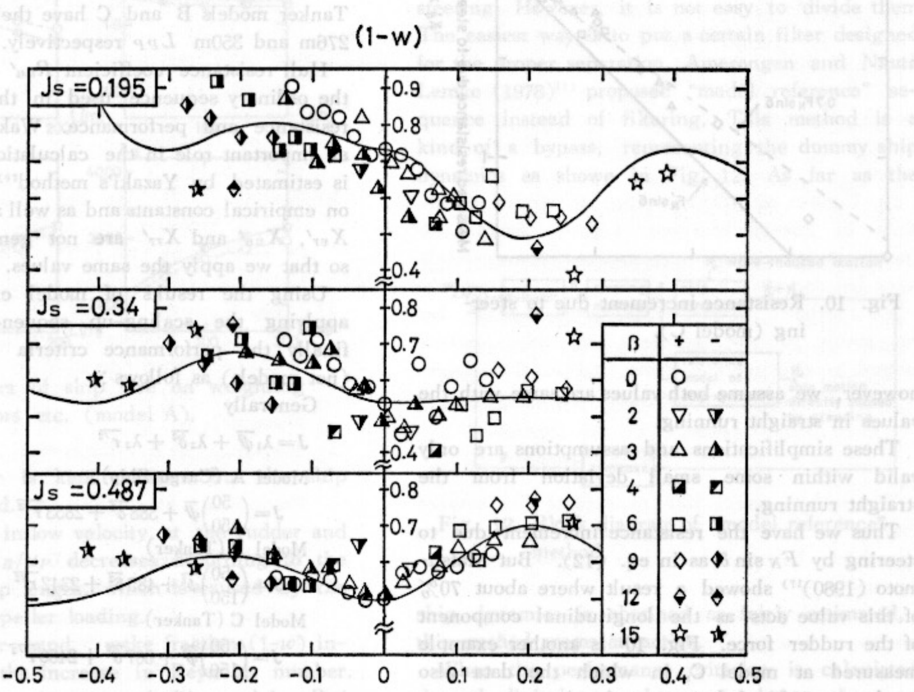

Fig. 8. Effect of ship motions on wake fraction (model C).

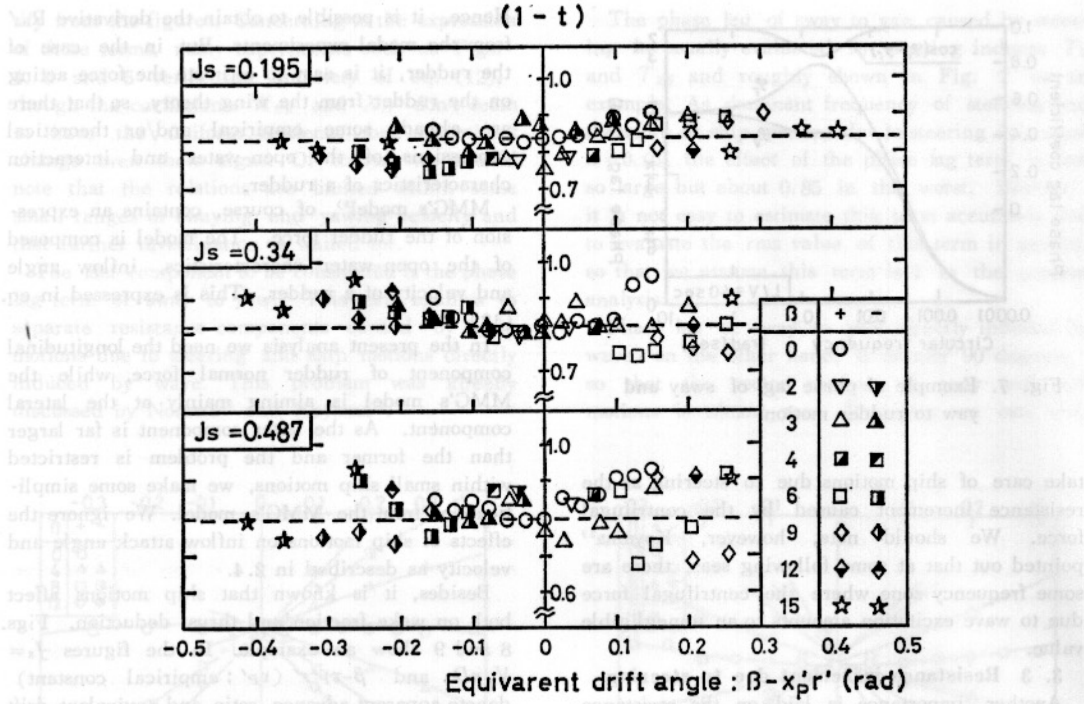

Fig. 9. Effect of ship motions on thrust deduction (model C).

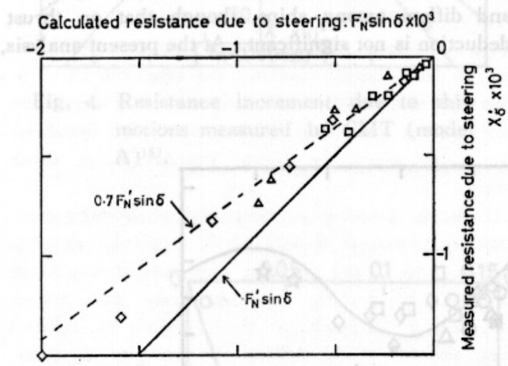

Fig. 10. Resistance increment due to steering (model C).

however, we assume both values are same with the values in straight running.

These simplifications and assumptions are only valid within some small deviation from the straight running.

Thus we have the resistance increment due to steering by $F_N \sin\delta$ as in eq. (12). But Matsumoto (1980)[17] showed a result where about 70% of this value acts as the longitudinal component of the rudder force. Fig. 10 is another example measured at model C, in which the data also reaches to 70% of the expected value.

3. 4 Performance criterion of real ships

By the procedure described above, the performance criterion of model ships is able to build up. But as we need the performance criterion of real ships, each derivative should be scaled up.

Model A is scaled up to 150m L_{PP} cargo ship. Tanker models B and C have their real ships of 276m and 350m L_{PP} respectively.

Hull resistance coefficient R_{uu}' is enlarged by the ordinary sequence used in the field of ship resistance and performance. Wake fraction plays an important role in the calculation of $R_{\delta\delta}'$ and is estimated by Yazaki's method[20]. Scale effects on empirical constants and as well as on derivatives X_{vr}', X_{vv}' and X_{rr}' are not generally known, so that we apply the same values.

Using the results of model experiments and applying the scaling-up sequence, we obtain finally the performance criteria of three ships (not models) as follows:

Generally

$$\left.\begin{aligned}
&J=\lambda_1\overline{\psi^2}+\lambda_2\overline{\delta^2}+\lambda_3\overline{r'^2}\\
&\text{Model A (Cargo Ship)}\\
&\quad J=\binom{50}{150}\overline{\psi^2}+388\,\overline{\delta^2}+2853\,\overline{r'^2}\\
&\text{Model B (Tanker)}\\
&\quad J=\binom{50}{150}\overline{\psi^2}+485\,\overline{\delta^2}+2342\,\overline{r'^2}\\
&\text{Model C (Tanker)}\\
&\quad J=\binom{50}{150}\overline{\psi^2}+607\,\overline{\delta^2}+2460\,\overline{r'^2}
\end{aligned}\right\} \quad \cdots\cdots(29)$$

where numbers in the parentheses correspond to the constraints of constant speed (upper row) and on-schedule (lower row) respectively.

It is noticed from the above equations that the weighting factor to rms value of rudder angle is roughly proportional to the fullness and the ratio of λ_2/λ_1 is about 8 to 14, if λ_1 is chosen as 50.

The variation of λ_3 is not significant, because R_{rr}' depends mainly upon the fullness of ships, while R_{uu}' depends also upon the size of ships. Effect of X_{vv}' and X_{rr}' on R_{rr}' is not so large that it seems profitable and allowable to apply $\frac{1}{2}(m'+X_{vr}'-Y_{\dot v}')$ for this term, when experimental data is not available. Because usually X_{vv}' and X_{rr}' are negative and phase lag term is less than 1, the error may be cancelled each other to some extent.

Fig. 11 shows the effect of the ship size on weighting factors etc., using the data of model A.

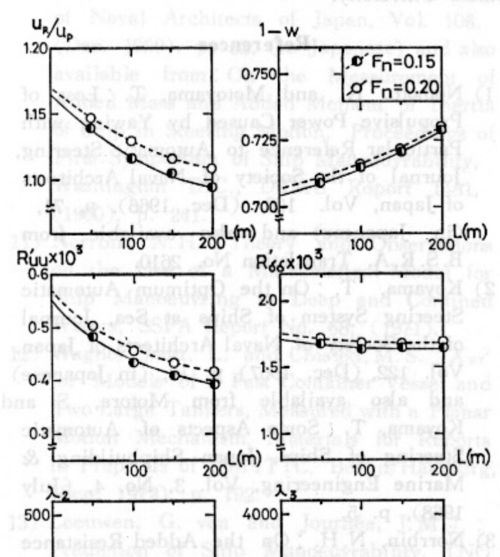

Fig. 11. Effect of ship size on weighting factors etc. (model A).

Froude number is kept constant but the ship length is altered.

The ratio of inflow velocity at the rudder and the propeller u_R/u_P decreases according to the increase in ship length, which is caused by the decrease in propeller loading.

On the other hand, wake fraction (1-w) increases due to the increase in Reynolds number. Consequently, in the case of this model, $R_{\delta\delta}'$ will keep nearly the same value. As R_{rr}' is considered free from scale effect, both λ_2 and λ_3 increase according to the decrease of R_{uu}'.

If the performance criterion of a similar ship is available, the criterion of the object ship can be modified by estimating the resistance coefficient R_{uu}' as a first approximation.

4. Applications and problems remained

4. 1 Separation of ship motions due to steering and wave

The performance criterion of autopilot steering is expected to be used in two ways. One is for design of ship forms or autopilots, and the other is on-line or off-line optimum adjustment of autopilots. In any cases it is necessary to separate ship motions due to steering and wave.

When data accumulation is achieved on board, measured signals of yawing deviation, rate of turn and rudder angle include both ship motions due to steering and wave. Wave-induced motion into yawing deviation should be included, because keeping the yawing deviation minimum itself is an important role of autopilots. Wave-induced motion into rudder motion should be excluded from the data, but most of the meaningless rudder movement excited by wave is filtered out by the electric filtering of autopilots and through the steering gear mechanism. In the case of rate of turn, wave-induced motion should be separated from the measured data, because most of the centrifugal force is caused by ship motions due to steering. However, it is not easy to divide them. The easiest way is to put a certain filter designed for the proper separation. Amerongen and Nauta Lemke (1978)[21] proposed "model reference" sequence instead of filtering. This method is a kind of a bypass, representing the dummy ship dynamics as shown in Fig. 12. As far as the

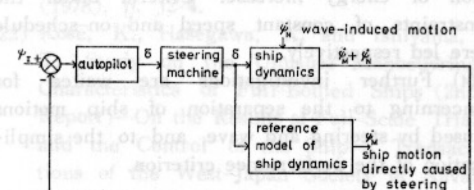

Fig. 12. Block diagram of "model reference" method.

ship dynamics is obtained or fairly estimated, this method seems attractive.

When the performance criterion is calculated through digital or analogue simulation, the same procedure is applicable more easily.

4. 2 Simplification of the performance criterion

It is not common to measure rate of turn directly on board. Direct measurement is possible by the rate gyro, but it is usually obtained by differentiating course signal. Differentiation itself is weak for noise and when a stepping motor is equipped as a compass repeater, the differentiation sometimes includes much noise than the signal. Hence, it is desirable to exclude the term of rate of turn from the criterion.

If the system stability is adequate enough, it is known that the centrifugal force due to yawing is negligible and in this case the criterion will be the same with Koyama's criterion[2]. But in case of an unstable ship or when a ship is yawing by self-sustaining oscillation, this term should be taken account of.

In the case of digital simulation, the rms value of rate of turn can be calculated using the response amplitude operator from the input spectrum of rudder angle. Time-based calculation is performed using the "model reference" method. But in these cases, the weighting factor is a function of steering frequency as well as ship configuration. Further investigations are necessary.

5. Conclusions

Main conclusions obtained in the present paper are:

(1) A performance criterion of autopilot navigation was derived considering the energy increase due to yawing.

(2) Each weighting factor is obtained from model experiments and reasonably estimated through the procedure described in the present paper.

(3) Certain constraint is needed for the evaluation of energy increase. Criteria under the constraints of constant speed and on-schedule were led respectively.

(4) Further investigations are waited for concerning to the separation of ship motions caused by steering and wave, and to the simplification of the performance criterion.

Acknowledgement

The author would like to express his sincere thanks to Prof. M. Nakato and Prof. K. Kose, Hiroshima University for their heartful support to his study.

The basic approach, calculation and experiments were carried out in Hiroshima University for the past several years. The author cannot forget the cooperated works performed by Prof. K. Kose, and former post-graduate students; Mr. N. Sasaki, Mr. T. Nakagami, Mr. T. Ishiyama and Mr. H. Umeda. Mr. T. Saeki and Mr. K. Ozaki are also appreciated for their collaboration in the model experiments. Especially the author is grateful to Prof. K. Kose who initiated the approach of the present work.

Prof. T. Koyama and Prof. M. Fujino, University of Tokyo kindly provided us the opportunity to use the facility; Manoeuvring and Seakeeping Model Basin.

The enthusiastic discussions between Prof. K. Nomoto, University of Osaka were reflected here and there in the present paper and Prof. T. Koyama advised the author in the basic but important aspect of the idea.

This work was suported by the Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture. Numerical calculations were carried out at the Computer Centre of Hiroshima University.

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