Theory of Ship Waves
(Wave-Body Interaction Theory)

By Professor Masashi KASHIWAGI

Problem Set: No.5
Date: June 09, 2021
Due: June 16, 2021

In the lecture on June 9 (and also in page 27 of the lecture note), it was shown that the velocity potential for the diffraction problem, $\varphi_D \equiv \varphi_0 + \varphi_4$, can be obtained from the following expression (integral equation):

$$C_P\,\varphi_D(\mathrm{P}) + \int_{S_H} \varphi_D(\mathrm{Q}) \frac{\partial}{\partial n_Q} G(\mathrm{P};\mathrm{Q})\, ds(\mathrm{Q}) = \varphi_0(\mathrm{P}) \tag{1}$$

where C_P denotes the solid angle (normalized with 2π) which is given as

$$C_P = \begin{cases} 1 & \text{when P is in the fluid} \\ \dfrac{1}{2} & \text{when P is on } S_H \end{cases} \tag{2}$$

Thus, when P is in the fluid region ($C_P = 1$), the scattering velocity potential $\varphi_4(\mathrm{P})$ can be written as

$$\varphi_4(\mathrm{P}) = -\int_{S_H} \varphi_D(\mathrm{Q}) \frac{\partial}{\partial n_Q} G(\mathrm{P};\mathrm{Q})\, ds(\mathrm{Q}) \tag{3}$$

There are some different ways for deriving Eq. (1). In fact, in the lecture note, Green's theorem is applied to the *exterior* fluid domain (surrounded by $S = S_H + S_F + S_{\pm\infty} + S_B$), and the body-hull boundary condition

$$\frac{\partial \varphi_D}{\partial n} = \frac{\partial}{\partial n}\big(\varphi_0 + \varphi_4\big) = 0 \quad \text{on } S_H$$

is taken into account.

As an alternative derivation method for Eq. (1), let us consider applying the Green's theorem to a combination of $\varphi_0(\mathrm{Q})$ and $G(\mathrm{P};\mathrm{Q})$ in the *interior* artificial fluid domain surrounded by the body surface (S_H) and the *interior* free surface (i.e. inside of a body). In this case, explain the derivation of the following equation

$$\big(C_P - 1\big)\varphi_0(\mathrm{P}) = \int_{S_H} \left\{ \frac{\partial \varphi_0(\mathrm{Q})}{\partial n_Q} - \varphi_0(\mathrm{Q}) \frac{\partial}{\partial n_Q} \right\} G(\mathrm{P};\mathrm{Q})\, ds(\mathrm{Q}) \tag{4}$$

and then prove correctness of Eq. (1). Then, explain why the Kochin function for the wave scattering problem can also be written as

$$\left.\begin{aligned} &\varphi_4(\mathrm{P}) \sim i H_4^{\pm}(K)\, e^{-Ky \mp iKx} \quad \text{as } x \to \pm\infty \\ &H_4^{\pm}(K) = -\int_{S_H} \varphi_D \frac{\partial}{\partial n}\, e^{-K\eta \pm iK\xi}\, ds(\xi,\eta) \end{aligned}\right\} \tag{5}$$

Hint: You should note that the positive normal direction in the Green's (or Gauss) theorem is outward from the interior fluid domain considered in the present case.