Theory of Ship Waves
(Wave-Body Interaction Theory)

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Problem Set: No.3
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1) In the same way as that in Quiz No. 5, let us derive the 3D Laplace equation in an orthogonal coordinate system O-$s_1 s_2 s_3$, specifically in the spherical coordinate system defined by

$$x = r\cos\theta, \quad y = r\sin\theta\cos\varphi, \quad z = r\sin\theta\sin\varphi. \tag{1}$$

First, confirm that the differential elements in the spherical coordinate system can be given by

$$\delta s_1 = \delta r, \quad \delta s_2 = r\,\delta\theta, \quad \delta s_3 = r\sin\theta\,\delta\varphi. \tag{2}$$

Then, obtain the 3D Laplace equation in the spherical coordinate system with consideration of mass conservation principle. (The process of derivation must be described.) The result must be of the following form:

$$\nabla^2\phi = \frac{\partial}{\partial r}\Big(r^2\frac{\partial\phi}{\partial r}\Big) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\theta\frac{\partial\phi}{\partial\theta}\Big) + \frac{1}{\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2} = 0 \tag{3}$$

2) From the result obtained above, derive the principal solution (which is independent of variables θ and φ, and hence a function of r only) of the velocity potential for the 3D point source *with unit strength*.

3) Prove that the linearized free-surface boundary condition for the case of a non-zero pressure distributed on the free surface can be given as follows:

$$\frac{\partial\phi}{\partial z} + K\phi = \frac{i\omega}{\rho g}\,\widetilde{p}\,(x,y) \equiv F(x,y) \quad \text{on } z=0 \tag{4}$$

where $K = \omega^2/g$, and $\phi(x,y,z)$ and $\widetilde{p}\,(x,y)$ are the spatial part of the velocity potential and the pressure distribution on the free surface, respectively, with the time-dependent part defined as $e^{i\omega t}$, like $\varPhi(\boldsymbol{x},t) = \mathrm{Re}\big[\,\phi(\boldsymbol{x})\,e^{i\omega t}\,\big]$.