Theory of Ship Waves
(Wave-Body Interaction Theory)

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Problem Set: No.2
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1) According to Eq. (1.29) in the text, the nonlinear free-surface boundary condition can be derived from the substantial derivative of the pressure being equal to zero. In fact, it is known that the 2nd-order free-surface boundary condition is given as

$$\frac{\partial^2 \Phi}{\partial t^2} - g\frac{\partial \Phi}{\partial z} + 2\nabla\Phi\cdot\nabla\frac{\partial \Phi}{\partial t} + \frac{1}{g}\frac{\partial \Phi}{\partial t}\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi}{\partial t^2} - g\frac{\partial \Phi}{\partial z}\right) = 0 \quad \text{on } z = 0 \tag{1}$$

This boundary condition is derived from Eq. (1.30) in the text by applying the Taylor-series expansion about $z = 0$. On the other hand, the same boundary condition must also be derived by combining the kinematic boundary condition

$$\frac{\partial \Phi}{\partial z} - \frac{\partial \zeta}{\partial t} - \nabla\Phi\cdot\nabla\zeta = 0 \quad \text{on } z = \zeta \tag{2}$$

and the dynamic boundary condition

$$\zeta = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi\right) \quad \text{on } z = \zeta. \tag{3}$$

Then, show that Eq. (1) can be obtained by eleminating ζ from Eq. (2) and Eq. (3), neglecting terms higher than and equal to $O(\Phi^3)$, and using the Taylor-series expansion about $z = 0$.
Hint: The Taylor-series expansion can be applied not only to the velocity potential itself, like Eq. (1.27) in the text, but also to the linear term in the boundary condition; that is, $\dfrac{\partial^2 \Phi}{\partial t^2} - g\dfrac{\partial \Phi}{\partial z}$ as a lump term.

2) Regarding the computation of time average of a product of two oscillatory quantities with the same circular frequency ω over one cycle of an oscillatory motion, the following calculation formula is available:

$$\frac{1}{T}\int_0^T \mathrm{Re}[\,A\,e^{i\omega t}]\mathrm{Re}[\,B\,e^{i\omega t}]\,dt = \frac{1}{2}\mathrm{Re}[\,A\,B^*] \tag{4}$$

where T is the period, A and B are of complex quantities in general, and B^* denotes the complex conjugate.
Show a proof of this formula, Eq. (4).