Theory of Ship Waves
(Wave-Body Interaction Theory)

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Problem Set: No.1
Date: April 14, 2021
Due: April 21, 2021

1) In deriving Euler's equation, the shear force due to the viscosity was not considered in the text, and thus the force acting on the fluid volume in the i-th direction was described as $-p\,n_i$. When the viscous shear is present, the corresponding force can be written as

$$\tau_{ij}\,n_j = \left\{ -p\,\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right\} n_j \tag{1}$$

where δ_{ij} is the Kronecker delta function, equal to 1 if $i = j$ and 0 if $i \neq j$, and μ is the coefficient of viscosity. Then by following the same procedure as in deriving (1.14) in the text, derive the Navier-Stokes equation.
The final result will be:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial^2 u_i}{\partial x_j^2} + g\,\delta_{i3} \tag{2}$$

2) Page 4 in the text says the normal vector can be obtained by the formula $\boldsymbol{n} = \nabla F/|\nabla F|$, provided that the body surface is represented by $F(x, y, z) = 0$. Confirm this relation for a (2-dimensional) ellipse and a (3-dimensional) prolate spheroid given by:

$$\left.\begin{aligned} x &= a\,\cos\theta \\ y &= a\epsilon\,\sin\theta\,\cos\varphi \\ z &= a\epsilon\,\sin\theta\,\sin\varphi\ , \quad \epsilon = b/a \end{aligned}\right\} \tag{3}$$

The result for a prolate spheroid will be given as follows:

$$n_1 = \epsilon\cos\theta/\Delta,\ \ n_2 = \sin\theta\cos\varphi/\Delta,\ \ n_3 = \sin\theta\sin\varphi/\Delta\,.$$

where

$$\Delta = \sqrt{\sin^2\theta + \epsilon^2\cos^2\theta}\ , \quad \epsilon = b/a\,.$$