Theory of Ship Waves (Wave-Body Interaction Theory)

Supplementary notes on Sections 3.1 and 3.2

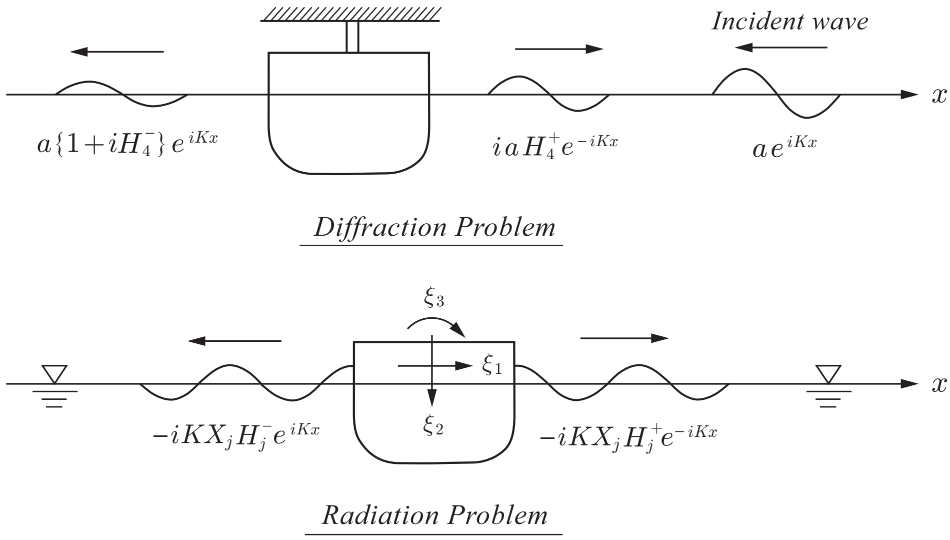

Fig.1 Schematic illustration for the diffraction and radiation problems.

Decomposition of the velocity potential

$$\phi(\boldsymbol{x}) = \frac{ga}{i\omega}\Big\{\varphi_0(\boldsymbol{x}) + \varphi_4(\boldsymbol{x})\Big\} + \sum_{j=1}^{3} i\omega X_j\,\varphi_j(\boldsymbol{x}) \tag{1}$$

$$\varphi_0(\boldsymbol{x}) = e^{-Ky+iKx}\,; \quad \text{which is given as input} \tag{2}$$

Body boundary condition

$$\frac{\partial\phi}{\partial n} = \sum_{j=1}^{3} i\omega X_j\,n_j \qquad (\,n_3 \equiv xn_2 - yn_1\,) \tag{3}$$

$$\longrightarrow \quad \frac{\partial}{\partial n}\big(\varphi_0 + \varphi_4\big) = 0\,, \quad \frac{\partial\varphi_j}{\partial n} = n_j \quad (j = 1,2,3) \tag{4}$$

Body-disturbance waves

$$\zeta(x) = \frac{i\omega}{g}\,\phi(x,0) = a\Big\{\varphi_0(x,0) + \varphi_4(x,0) - K\sum_{j=1}^{3}\frac{X_j}{a}\,\varphi_j(x,0)\Big\} \tag{5}$$

$$\varphi_j(x,y) \sim iH_j^{\pm}(K)\,e^{-Ky\mp iKx} \quad \text{as } x\to\pm\infty \quad (j = 1 \sim 4) \tag{6}$$

$$\text{Radiation wave:} \quad \zeta_j^{\pm} = -iKX_jH_j^{\pm}(K)\ \big(\ \equiv X_j\bar{A}_j\,e^{i\varepsilon_j^{\pm}}\ \big) \tag{7}$$

$$\text{Scattered wave:} \quad \zeta_4^{\pm} = iaH_4^{\pm}(K) \tag{8}$$

$$\zeta(x) = a\Big[\,iH_4^{+}(K) - iK\sum_{j=1}^{3}\frac{X_j}{a}H_j^{+}(K)\Big]\,e^{-iKx} \quad (\textit{propagating to positive}) \tag{9}$$

$$+a\Big[\,1 + iH_4^{-}(K) - iK\sum_{j=1}^{3}\frac{X_j}{a}H_j^{-}(K)\Big]\,e^{+iKx} \quad (\textit{propagating to negative}) \tag{10}$$

Case (1)

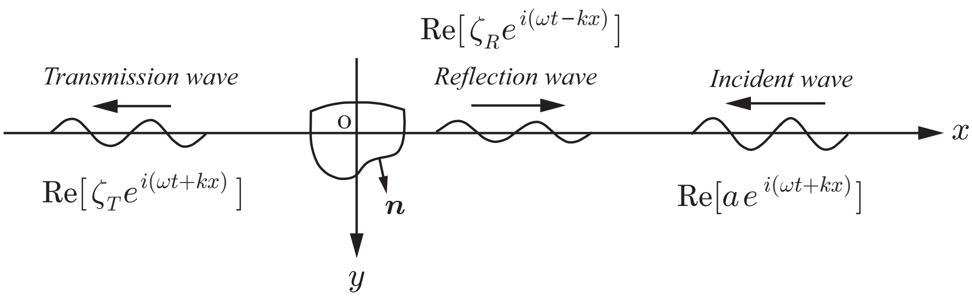

Fig.2 Case of incident wave incoming from the positive x-axis

$$\zeta_R\, e^{-iKx} = a\Big\{\, iH_4^+(K) - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^+(K)\,\Big\}\, e^{-iKx} \tag{11}$$

$$\longrightarrow \quad C_R \equiv \frac{\zeta_R}{a} = \underbrace{iH_4^+(K)}_{\equiv R} - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^+(K) \tag{12}$$

$$R = iH_4^+(K) \tag{13}$$

$$\zeta_T\, e^{+iKx} = a\Big\{\, 1 + iH_4^-(K) - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^-(K)\,\Big\}\, e^{+iKx} \tag{14}$$

$$\longrightarrow \quad C_T \equiv \frac{\zeta_T}{a} = \underbrace{1 + iH_4^-(K)}_{\equiv T} - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^-(K) \tag{15}$$

$$T = 1 + iH_4^-(K) \tag{16}$$

Case (2)

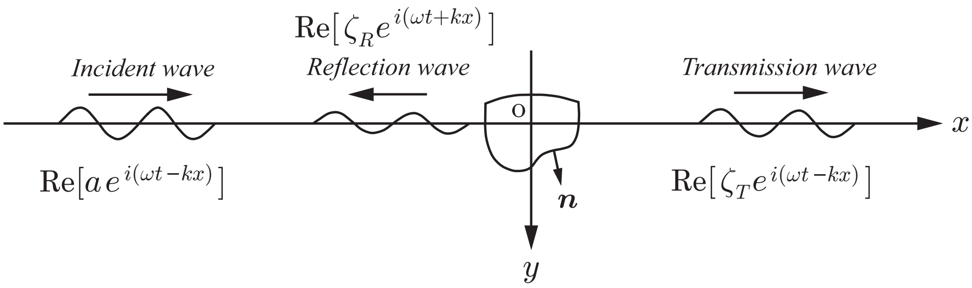

Fig.3 Case of incident wave incoming from the negative x-axis

$$C_R \equiv \frac{\zeta_R}{a} = \underbrace{ih_4^-(K)}_{\equiv R} - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^-(K) \tag{17}$$

$$R = ih_4^-(K) \tag{18}$$

$$C_T \equiv \frac{\zeta_T}{a} = \underbrace{1 + ih_4^+(K)}_{\equiv T} - iK\sum_{j=1}^{3}\frac{X_j}{a}\, H_j^+(K) \tag{19}$$

$$T = 1 + ih_4^+(K) \tag{20}$$

Hydrodynamic Relations Derived with Green's Theorem

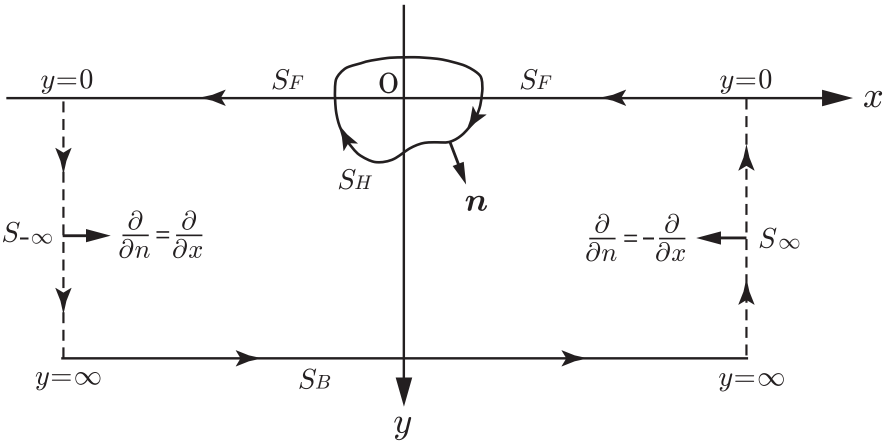

Fig. 3.4 Application of Green's theorem

$$\int_{S_H}\left(\phi\frac{\partial\psi}{\partial n}-\psi\frac{\partial\phi}{\partial n}\right)d\ell=\frac{1}{2K}\left[\left(\phi\frac{\partial\psi}{\partial x}-\psi\frac{\partial\phi}{\partial x}\right)_{y=0}\right]_{x=-\infty}^{x=+\infty} \tag{1}$$

Table 1 Summary of Some Important Hydrodynamic Relations

	ϕ	ψ	Relations to be obtained
1	φ_i	φ_j	$A_{ij}=A_{ji}\,,\ B_{ij}=B_{ji}$: Symmetry relations in the radiation forces
2	φ_i	$\overline{\varphi}_j$	$B_{ij}=\dfrac{1}{2}\,\rho\omega\big\{\,H_i^+\overline{H}_j^+ + H_i^-\overline{H}_j^-\,\big\}$ Energy conservation in the radiation problem
3	ϕ_D	$\overline{\phi}_D$	$\lvert R\rvert^2+\lvert T\rvert^2=1$ Energy conservation in the diffraction problem
4	ϕ_D	φ_j	$E_j=\rho ga\,H_j^+$ Haskind-Hanaoka-Newman's relation
5	ϕ_D	$\overline{\varphi}_j$	$E_j=\rho ga\big\{\,\overline{H}_j^+R+\overline{H}_j^-T\,\big\}$
			Relations in 4 and 5 gives $H_j^+=\overline{H}_j^+R+\overline{H}_j^-T,\quad (\,R=iH_4^+,\ T=1+iH_4^-\,)$ For a symmetry body $H_4^\pm=i\,e^{i\varepsilon_H}\cos\varepsilon_H\mp e^{i\varepsilon_S}\sin\varepsilon_S$ Furthermore $\begin{cases} H_3^+=H_1^+\,\ell_w & (\,\ell_w \text{ is real quantity; the phase is the same)} \\ B_{13}=B_{31}=\ell_w\,B_{11} \\ B_{33}=\ell_w^2\,B_{11} & \text{(Bessho's relation)}\end{cases}$
6	ϕ_D	ψ_D	$h_4^+=H_4^-$ Transmission wave (both amplitude & phase) past an asymmetric body is the same irrespective of the direction of incident wave (Bessho's relation)
7	$\overline{\phi}_D$	ψ_D	$h_4^-=\overline{H}_4^+\dfrac{1+i\,H_4^-}{1-i\,\overline{H}_4^-}$ The amplitude of reflection wave by an asymmetric body is also the same irrespective of the direction of incident wave (Bessho's relation)
			Relations of 3 and 7 for a symmetric body gives $\lvert R\pm T\rvert=1$: Wave energy equally splitting law

In the above, $\phi_D=\varphi_0+\varphi_4$ and $\overline{\phi}$ denotes the complex conugate of ϕ.