Wave-energy Absorption by a Rotating Pendulum-type Cylinder inside a Floating Body

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Ocean energy

Enormous and renewable
Less negative influence on the environment

Background

Wave energy absorption efficiency

Generally

In limited wave frequency range
- Rectangular floating body
- Particular shape floating body
- Sway, heave and roll coupled motion

Presentation programs

Only roll motion
Rectangular floating body

Electric power generator

No sliding
Roll motion equation of a floating body

\[ I_0 \ddot{\phi} + A_{44} \ddot{\phi} + B_{44} \dot{\phi} + (M + m) \overline{GM} \sin \theta - FR = M_4 \quad (1) \]

Motion equations of a smaller cylinder

\[
\begin{cases}
\text{Tangential direction} & m(R - r)\ddot{\theta} = -mg \sin \theta + F \quad (2) \\
\text{Rotational direction} & I_C \ddot{\psi} = -N \dot{\psi} - Fr \quad (3)
\end{cases}
\]

Condition of no sliding between a floating body and a smaller cylinder

\[
(R - r) \dot{\theta} - r \dot{\psi} \equiv -R \dot{\phi} \quad (4)
\]

Velocity of a smaller cylinder Velocity of the circular surface
Linearization in terms of $\phi$ and $\theta$

\[
\begin{align*}
\left[-\omega^2 I_A + (M + m) gGM + i\omega \left( B_{44} + N \frac{R^2}{r^2} \right) \right] & \Phi \\
+ \frac{R(R + r)}{r^2} \left[-\omega^2 I_C + i\omega N \right] & \Theta = E_4
\end{align*}
\]

(5)

\[
\begin{align*}
[ -\omega^2 I_C + i\omega N ] & \Phi + \left[ -\omega^2 I + mg \frac{r^2}{R - r} + i\omega N \right] \Theta e^{i\omega t} = 0
\end{align*}
\]

(6)

Complex amplitudes

$\phi = \text{Re} \left[ \Phi e^{i\omega t} \right]$

$\theta = \text{Re} \left[ \Theta e^{i\omega t} \right]$

where

$I_A \equiv I_0 + A_{44} + I_C \frac{R^2}{r^2}$, \quad $I \equiv I_C + m r^2$

(7)

Transformation of equations
Transformation of equations

\[(M + m) \overline{GM} - \omega^2 I_A \equiv \rho \omega^2 R^4 P^2\]

Restoring and inertia terms
\[\omega^2 I_C \equiv \rho \omega^2 R^2 r^2 Q^2\]
\[mg \frac{r^2}{R - r} - \omega^2 I \equiv \rho \omega^2 R^2 r^2 S^2\]

Haskind relation
\[B_{44} = \rho \omega R^4 |H_4^+|^2 \equiv \rho \omega R^4 h\]

Energy conservation
\[E_4 = \rho g \zeta_0 R^2 H_4^+\]

where \[h \equiv |H_4^+|^2\]

Kochin function in the radiation wave

Damping coefficient of electric-power generator
\[N \frac{R^2}{r^2} \equiv \beta B_{44} = \rho \omega R^4 \beta h\]

Proportional coefficient (Real number)
Amplitudes of floating body and inner cylinder

\[
\left\{ P^2 + ih(1 + \beta) \right\} \frac{\Phi}{K\zeta_a} + \left\{ -Q^2 + ih\beta \right\} \frac{R - r}{R} \frac{\Theta}{K\zeta_a} = \frac{H_4^+}{(K\zeta_a)^2} \tag{11}
\]

\[
\left\{ -Q^2 + ih\beta \right\} \frac{\Phi}{K\zeta_a} + \left\{ S^2 + ih\beta \right\} \frac{R - r}{R} \frac{\Theta}{K\zeta_a} = 0 \tag{12}
\]

where

\[
\Delta = P^2 S^2 - Q^4 - \beta h^2 + ih\left\{ \beta (P^2 + S^2 + 2Q^2) + S^2 \right\} \tag{14}
\]
Power of regular incident wave

\[ P_W = \frac{1}{2} \rho g \zeta_a^2 \left( \frac{g}{2\omega} \right) = \frac{\rho g^2 \zeta_a^2}{4\omega} \quad (15) \]

Power of electric power generator

\[ a \equiv P^2 + Q^2, \quad b \equiv \frac{1}{T} \int_0^T \omega^2 dt \]
\[ c \equiv a + b = P^2 + S^2 + 2Q^2, \quad d \equiv S^2 \]
\[ (\Phi) \equiv P^2 \rightarrow Q^2/\omega^2 \equiv \frac{R}{r} \phi \equiv \frac{R^2 - r}{r} \dot{\theta} \]

Notations

(16) \[ P_E = \frac{1}{2} N \frac{R^2}{r^2} \omega^2 \left| \Phi + \frac{R - r}{R} \Theta \right|^2 \quad (18) \]
Wave-energy absorption efficiency

\[ \eta \equiv \frac{P_E}{P_W} = 2\beta h^2 \left| \frac{b}{\Delta} \right|^2 = \frac{2\beta h^2 b^2}{|\Delta|^2} \]  

(19)

where

\[ |\Delta|^2 = (de - \beta h^2)^2 + h^2(\beta c + d)^2 \]

Differentiating \( \eta \) with respect to \( \beta \)

\[ \frac{d\eta}{d\beta} = 0 \quad \Rightarrow \quad \beta^2 = \frac{d^2(h^2 + e^2)}{h^2(c^2 + h^2)} \]  

(20)

Maximum absorption efficiency

\[ \eta_{\text{max}} = \frac{1}{1 + \frac{d^2(h^2 + e^2)}{\beta h^2 b^2}} \]  

(21)
Substituting the amplitude of floating body into (23)

**Symmetric wave component**

\[ \mathcal{A} = \frac{1}{2} (C_R + C_T) = \frac{1}{2} \frac{H_3^+}{H_3^+} - iKR \left( \frac{Y}{\zeta_\alpha} \right) H_3^+ \]  

**Anti-symmetric wave component**

\[ \mathcal{B} = \frac{1}{2} (C_R - C_T) = \frac{1}{2} \frac{H_4^+}{H_4^+} - i (KR)^2 \left( \frac{\Phi}{K \zeta_\alpha} \right) H_4^+ \]  

**Reflection and transmission waves**

Substituting the amplitude of floating body \( \Phi / K \zeta_\alpha \) into (23)

\[ \mathcal{B} = \frac{1}{2} \frac{H_4^+}{H_4^+} - i \left( H_4^+ \right)^2 \frac{d + i h}{\Delta} \]

\[ = \frac{1}{2} \frac{H_4^+}{H_4^+} de + \beta h^2 + ih (\beta c - d) \]

\[ = \frac{1}{2} \frac{H_4^+}{H_4^+} de - \beta h^2 + ih (\beta c + d) \]  

\[ \text{(24)} \]
Perfect absorption of anti-symmetric wave component

\[ \| \]

Anti-symmetric wave component \( B = 0 \)

\[
\beta = \frac{d (h + i e)}{h (c - i h)} \quad \Rightarrow \quad \beta \overline{\beta} = \beta^2 = \frac{d^2 (h^2 + e^2)}{h^2 (h^2 + c^2)} = (20)
\]

Real number

Conditions of maximum absorption efficiency

\[ ec + h^2 = 0 \ , \ \beta = \frac{d}{c} > 0 \quad (25) \]

Maximum absorption efficiency

\[ \eta_{max} = \frac{1}{2} \quad (26) \]
\[ \begin{align*}
\{ P^2 + \sigma (1 + \beta) \} \frac{\Phi}{K \zeta_a} \\
+ \{ -Q^2 + \sigma \beta \} \frac{R - r}{R} \frac{\Theta}{K \zeta_a} = \frac{H_+}{(KR)^2} \\
\{ -Q^2 + \sigma \beta \} \frac{\Phi}{K \zeta_a} \\
+ \{ S^2 + \sigma \beta \} \frac{R - r}{R} \frac{\Theta}{K \zeta_a} = 0 \\
\beta - \frac{d}{c} = 0 \\
e c + h^2 = 0
\end{align*} \]

(25)

Parameters
\[
\begin{align*}
\kappa &= 0.50 \\
r &= 0.10 \\
GM &= 0.20 \\
M/m &= 4.00 \\
\beta &= 1.30
\end{align*}
\]
\[ \mathcal{B} = \frac{1}{2} |C_R - C_T| \]

\[ \eta = \frac{P_E}{P_W} = \frac{2\beta h^2 b^2}{|\Delta|^2} \]

\[ \frac{\beta}{c} - \frac{d}{c} = 0 \]

\[ ec + h^2 = 0 \]

\[ \mathcal{B} = 0 \]

\[ \eta_{max} = \frac{1}{2} \]
Incident wave and reflection wave

Particular body shape

No wave

Electric power generator
Haskind relation

\[ B_{44} = \rho \omega R^4 \left| H_4^+ \right|^2 \equiv \rho \omega R^4 h \]

\[ B_{44} = \frac{1}{2} \rho \omega R^4 \left\{ H_4^+ \overline{H_4^+} + H_4^- \overline{H_4^-} \right\} \equiv \frac{1}{2} \rho \omega R^4 h \]  \hspace{1cm} (27)

No transmission wave

\[ \left< H_4^- \right> = \overline{H_4^-} = 0 \]  \hspace{1cm} (28)

Bessho-Newman relation

\[ H_4^+ = \overline{H_4^+} R + \overline{H_4^-} T = \overline{H_4^+} R \]

\[ |R| = 1 \]  \hspace{1cm} (29)

Energy conservation

\[ |R|^2 + |T|^2 = 1 \]

\[ |T| = 0 \]  \hspace{1cm} (30)
Amplitude of floating body (no transmission wave)

\[
C_R = R - i \left( KR \right)^2 \left( \frac{\Phi}{K \zeta_a} \right) H_4^+
\]

\[
C_T = T - i \left( KR \right)^2 \left( \frac{\Phi}{K \zeta_a} \right) H_4^-
\]

\[
C_R = \frac{H_4^+}{H_4^-} \frac{de + \frac{1}{4} \beta h^2 + i \frac{1}{2} h (\beta c - d)}{de - \frac{1}{4} \beta h^2 + i \frac{1}{2} h (\beta c - d)}
\]

\[
C_T = 0
\]
Perfect absorption of reflection wave

\[ C_R = \frac{H_4^+}{H_4^-} \frac{d e + \frac{1}{4} \beta h^2 + i\frac{1}{2} h (\beta c - d)}{d e - \frac{1}{4} \beta h^2 + i\frac{1}{2} h (\beta c - d)} = 0 \]

\[ \beta = \frac{2d (ih - 2e)}{h (h + i2c)} \]  \( \text{Real number} \)  \( (35) \)

Conditions of maximum absorption efficiency

\[ 4ec + h^2 = 0, \quad \beta = \frac{d}{c} \]  \( (36) \)

Maximum absorption efficiency (no transmission wave)

\[ \eta_{max} = \frac{2}{1 + \frac{d^2(h^2 + 4e^2)}{\beta h^2 b^2}} = 1 \]  \( (37) \)
Symmetric floating body

Asymmetric floating body

Sway, heave and roll

Asymmetric motion

Maximum absorption efficiency

Asymmetric and symmetric motion

Maximum absorption efficiency
Sway
\[(M + A_{22}) \ddot{x} + B_{22} \dot{x} + A_{23} \ddot{y} + B_{23} \dot{y} + A_{24} \dddot{\phi} + B_{24} \ddot{\phi} + F \cos \theta - T \sin \theta = F_2 \quad (38)\]

Heave
\[A_{32} \ddot{x} + B_{32} \dot{x} + (M + A_{33}) \ddot{y} + B_{33} \dot{y} + \rho g A_w y + A_{34} \dddot{\phi} + B_{34} \ddot{\phi} - F \sin \theta - T \cos \theta = F_3 \quad (39)\]

Roll
\[A_{42} \ddot{x} + B_{42} \dot{x} + A_{43} \ddot{y} + B_{43} \dot{y} + (I_{ZZ} + A_{44}) \dddot{\phi} + B_{44} \ddot{\phi} + (m + M) g \overline{GM} \sin \phi - FR = F_4 \quad (40)\]
Absorption efficiency

Wave amplitudes

\[ A = \frac{|C_R + C_T|}{2} \]

\[ B = \frac{|C_R - C_T|}{2} \]
Numerical results of asymmetric body

Absorption efficiency

Wave amplitudes

$|C_R|$

$|C_T|$
Future works

- Numerical calculation of sway, heave and roll coupled motion
- Calculation in time domain
- Calculation of 3D model
- Considering nonlinear effect
- Calculate appropriate body shape
Thank you
Model experiment

Aims of this experiment

- Real motion
- Appropriateness of the present theory (Linear theory, only roll motion)
- No electric-power generator
- Damping coefficient of smaller cylinder
- the resistance of electric-power generator
Radius of interior circular surface: 15.0 cm
Radius of inner smaller cylinder: 3.2 cm
Displacement: 185.9 N
Weight of smaller cylinder: 4.9 N
Draft: 17.8 cm
Width: 29.7 cm
Breadth: 37.5 cm
Gyrational radius: 8.2 cm
Incident wave periods
From 0.5 sec to 1.8 sec with interval of 0.1 sec
(0.75 sec and 0.85 sec were added to see phenomena around
the resonant frequency of the smaller inner circular cylinder)
Around resonant frequency of the floating body
Around resonant frequency of the inner cylinder
Motion amplitude of model experiment

- Using linear wave damping
- Considering only roll motion
- Friction between body and the wall of water channel

Difference between numerical results and measured results of floating body
Absorption efficiency of model experiment

Damping coefficient of smaller cylinder $N$ is very small

$N = 2.7 \times 10^{-4}$ kgm$^2$/sec