



# An Investigation on Ship Parametric Rolling Based on Time Domain Strip Theory

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# Outline

- A Brief Introduction on Ship Parametric Rolling
- An Enhanced Time Domain Strip Theory
- Numerical Simulation and Experiment Validation
- Summary

# A Brief Introduction on Ship Parametric Rolling



The APL China incident, 1998 ([cargolaw.com](http://cargolaw.com))



Container ship motion in waves ([youtube.com](https://www.youtube.com))



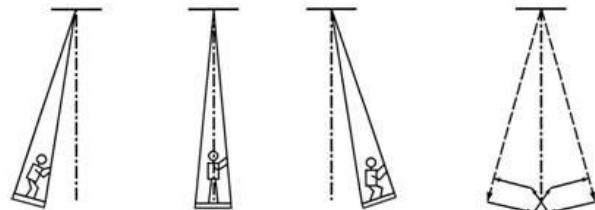
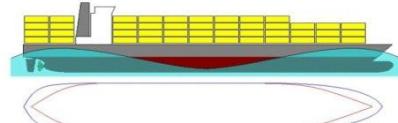
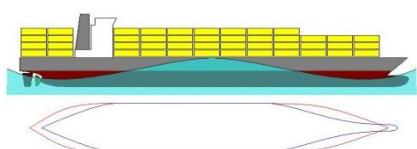
Cruise ship **Voyager** suffered from parametric rolling, 2005  
([youtube.com](https://www.youtube.com))



Parametric rolling model test conducted at HEU  
SOE , Osaka, December, 2013

# A Brief Introduction on Ship Parametric Rolling

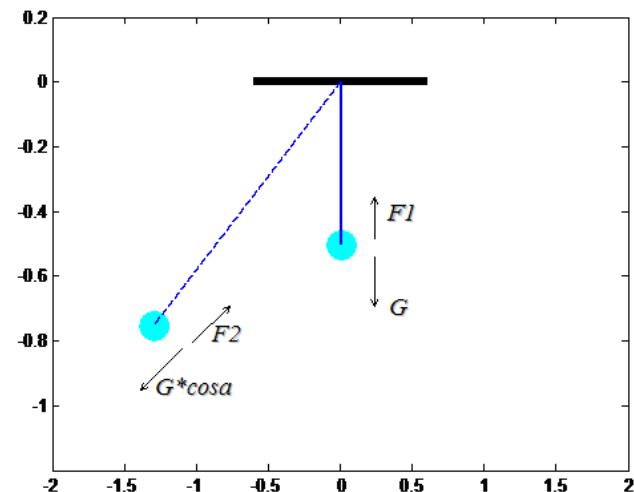
Parametric rolling occurring for any ship having large righting arm variations between trough and crest condition.



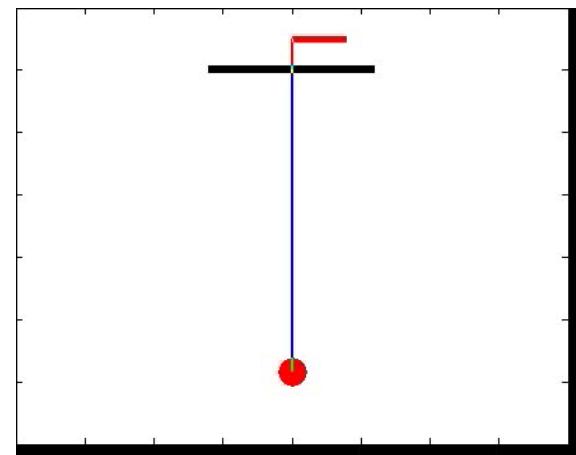
Ship on the wave crest and trough.(From STAB Webpage)

$$GM = \frac{I}{\nabla}$$

Swing model



Physical explain of parametric excitation



The motion of the pendulum  
Disturbance frequency equals to two times the natural frequency

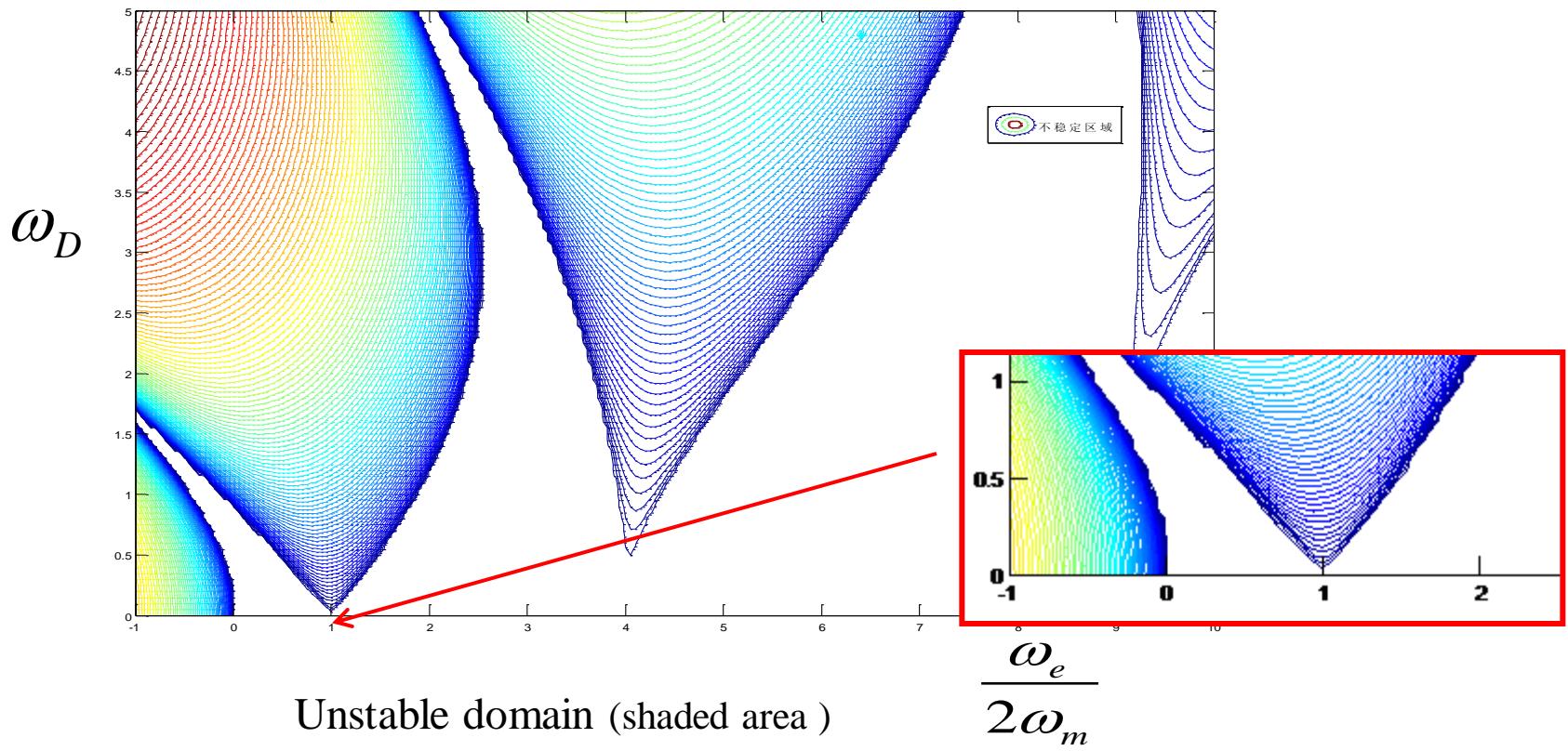
# A Brief Introduction on Ship Parametric Rolling

$$\ddot{\phi} + \cancel{\dot{\phi}} + (\omega_m^2 + \omega_D^2 \cos(\omega_e t)) \cdot \phi = 0$$

Mathieu Equation

$\omega_m$  The natural frequency       $\omega_D$  Amplitude of the disturbance

$\omega_e$  The encounter frequency



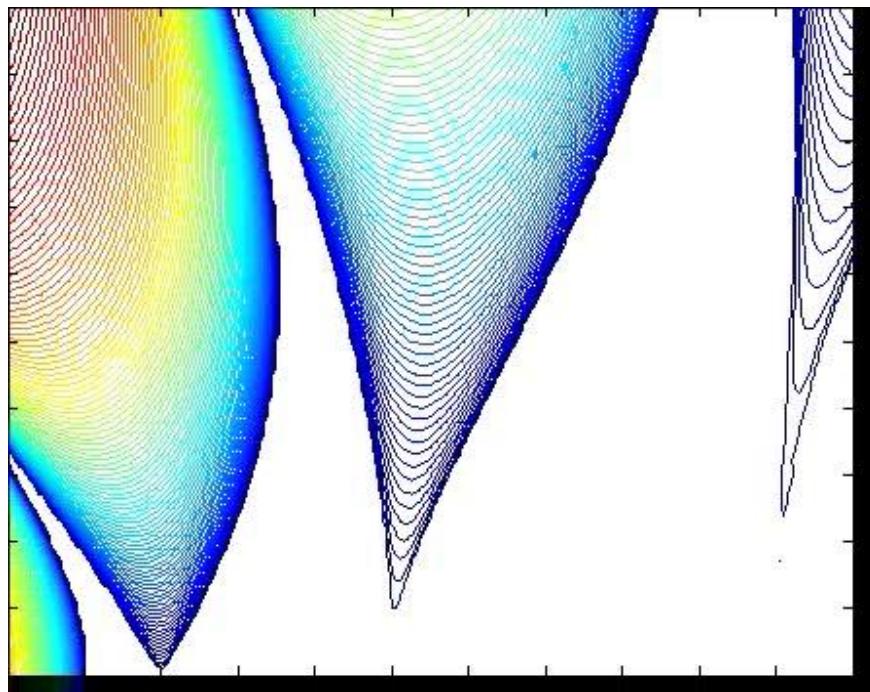


# A Brief Introduction on Ship Parametric Rolling

$$\ddot{\phi} + 2\delta\dot{\phi} + (\omega_m^2 + \omega_D^2 \cos(\omega_e t)) \cdot \phi = 0$$

$\omega_m$  The natural frequency       $\omega_D$  Amplitude of the disturbance

$\omega_e$  The encounter frequency



The unstable domain vary with different roll damping level



# A Brief Introduction on Ship Parametric Rolling

## Factors affecting the accuracy of numerical simulation:

- Parameters contributed to righting arm variations
  - Waves
  - Unsteady motion coupling
- The exact roll damping model
  - Waves
  - Unsteady motion coupling
- Large amplitude hydrodynamics model



# An Enhanced Time Domain Strip Theory

Motion Equation:

$$A\ddot{\eta} = F^D(t) + F^R(t) + F^I(t) + F^S(t) + F^\nu$$

$F^R$  Radiation Force

$F^D$  Diffraction Force

$F^I$  Wave excitation force

$F^S$  Restoring Force

$F^\nu$  Viscous force



# An Enhanced Time Domain Strip Theory

Time domain radiation force:

$$F^R_{jk}(t) = -\mu_{jk} \ddot{\eta}_k(t) - b_{jk} \dot{\eta}_k(t) - c_{jk} \eta_k(t) - \int_0^t K_{jk}(t-\tau) \dot{\eta}_k(\tau) d\tau \quad \text{Cummins (1962)}$$

$$\begin{cases} \nabla^2 \varphi_k, \nabla^2 \psi_k = 0 \\ \varphi_k, \psi_k = 0 \quad (\text{on } S_F) \\ \frac{\partial \varphi_k}{\partial n} = n_k \quad (\text{on } S_0) \\ \frac{\partial \psi_k}{\partial n} = m_k \quad (\text{on } S_0) \\ [\varphi_k, \nabla \psi_k] \rightarrow 0 \quad (r \rightarrow \infty) \end{cases}$$

$$\begin{cases} \nabla^2 \bar{\chi}_k = 0 \\ [(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x})^2 + g \frac{\partial}{\partial z}] (\bar{\chi}_k + \psi_k) = 0 \quad (\text{on } S_F) \\ \frac{\partial \bar{\chi}_k}{\partial n} = 0 \quad (\text{on } S_0) \\ \bar{\chi}_k|_{t=0} = 0 \quad \frac{\partial \bar{\chi}_k}{\partial t}|_{t=0} = -g \frac{\partial \varphi_k}{\partial z} \quad (\text{on } S_F) \end{cases}$$

$$\begin{cases} \mu_{jk} = \rho \iint_{S_0} \varphi_k n_j ds \\ b_{jk} = \rho \left\{ \iint_{S_0} \psi_k n_j ds - \iint_{S_0} \varphi_k m_j ds \right\} \\ c_{jk} = -\rho \iint_{S_0} \psi_k m_j ds \\ K_{jk}(t) = \rho \left\{ \iint_{S_0} \frac{\partial}{\partial t} \bar{\chi}_k(p, t) n_j ds - \iint_{S_0} \bar{\chi}_k(p, t) m_j ds \right\} \end{cases}$$

Frequency domain radiation force:

$$F_{jk}(t) = \operatorname{Re}\{e^{i\omega_e t} [\omega_e^2 A_{jk} - i\omega_e B_{jk}]\}$$

$$K_{jk}(\tau) = \frac{2}{\pi} \int_0^\infty (B_{jk}(\omega_e) - b_{jk}) \cos \omega_e \tau d\omega_e$$

Kramers-Kronig relation(Cummins,1962,Ogilvie,1964)

Bradley, 1987

Liapis (1986)

$$b_{jk} \equiv 0 \quad \times$$

$$b_{jk} \equiv 0 \quad j = k$$

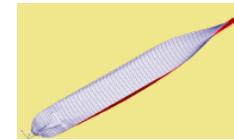
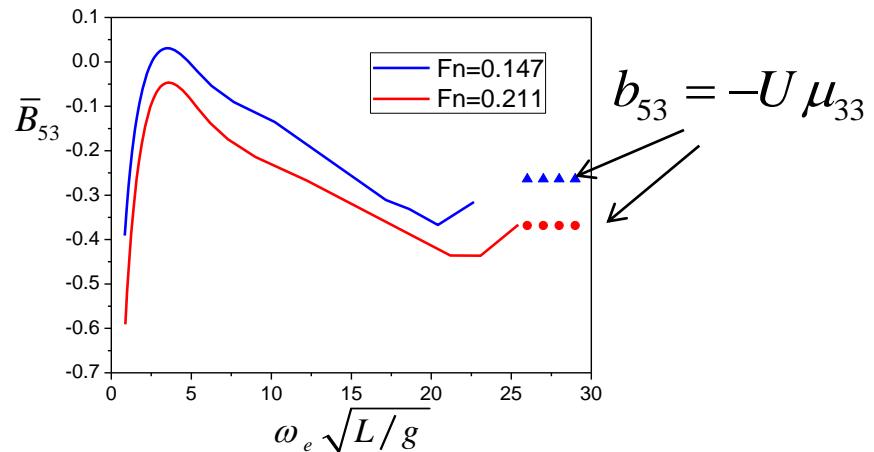
$$b_{35} = -b_{53} = U \mu_{33}$$

$$b_{26} = -b_{62} = -U \mu_{22}$$

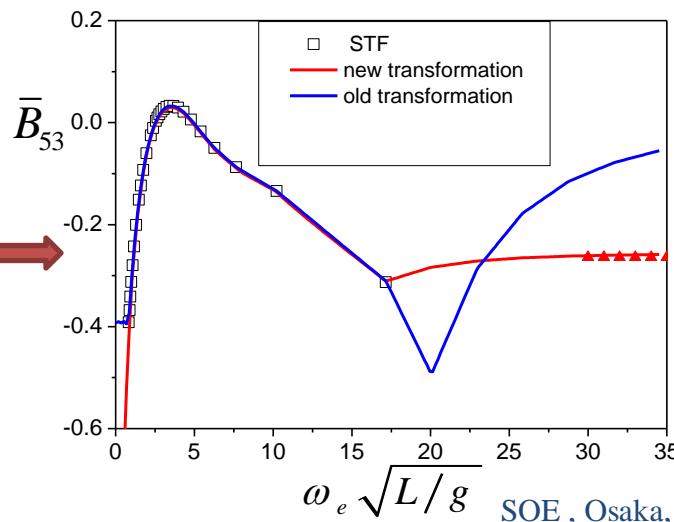
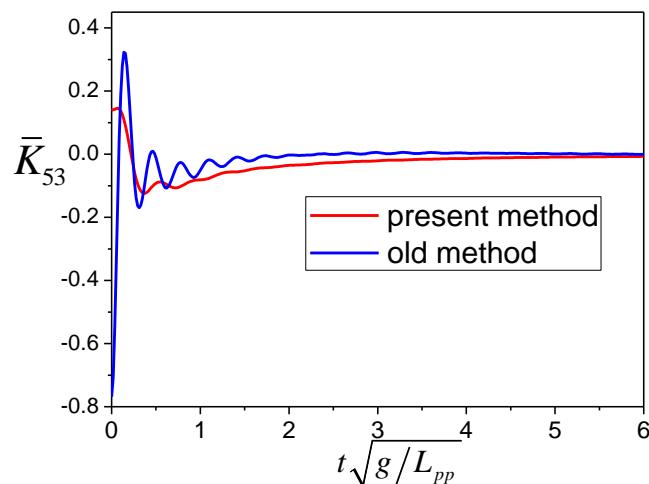


## Some corrections for $b_{jk}$ term

$$K_{jk}(\tau) = \frac{2}{\pi} \int_0^\infty (B_{jk}(\omega_e) - b_{jk}) \cos \omega_e \tau d\omega_e$$

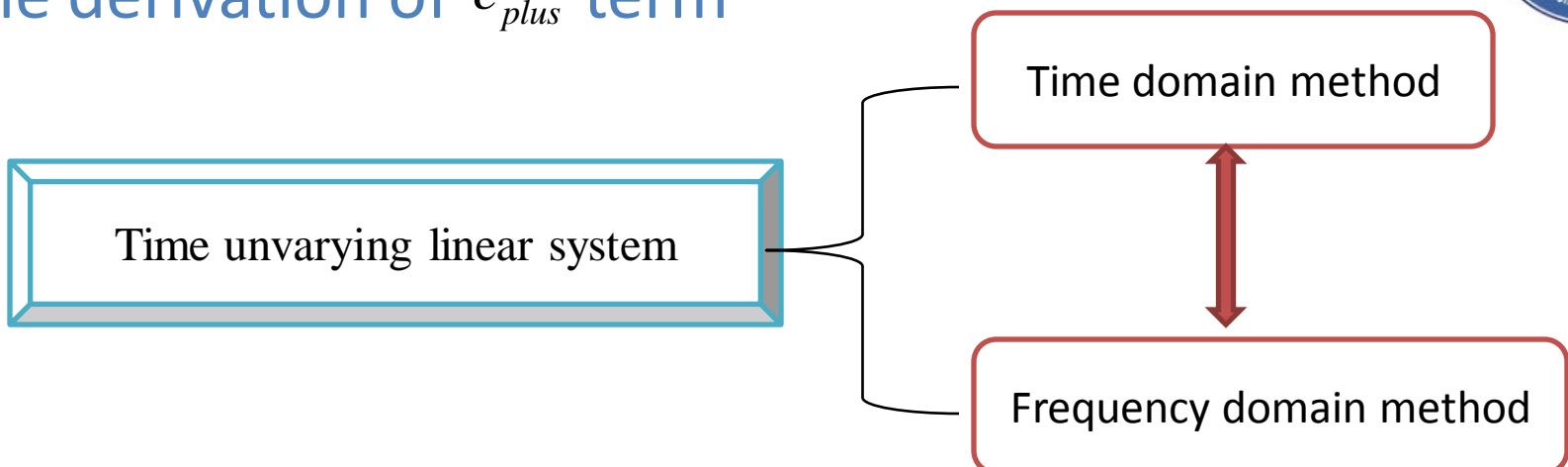


$$b_{jk} = \lim_{\omega \rightarrow \infty} B_{jk}(\omega_e) \quad j \neq k$$





# The derivation of $c_{plus}$ term



$$B_{jk}(\omega) = \frac{2}{\pi} P.V. \int_0^{\infty} \frac{\omega_1^2}{\omega_1^2 - \omega^2} [\mu_{jk} - A_{jk}(\omega_1)] d\omega_1$$

$$\mu_{jk} - A_{jk}(\omega) = -\frac{2}{\pi} P.V. \int_0^{\infty} \frac{B_{jk}(\omega_1)}{\omega_1^2 - \omega^2} d\omega_1$$

$$j, k = 1 \dots 6$$

$$U = 0$$

$$U \neq 0$$

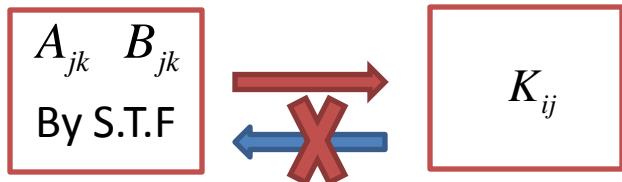
**S.T.F** is not a complete  
Theory to handle ship motion  
with forward speed  
(zero speed approximation)

Kramers-Kronig relation(Cummins,1962,Ogilvie,1964)

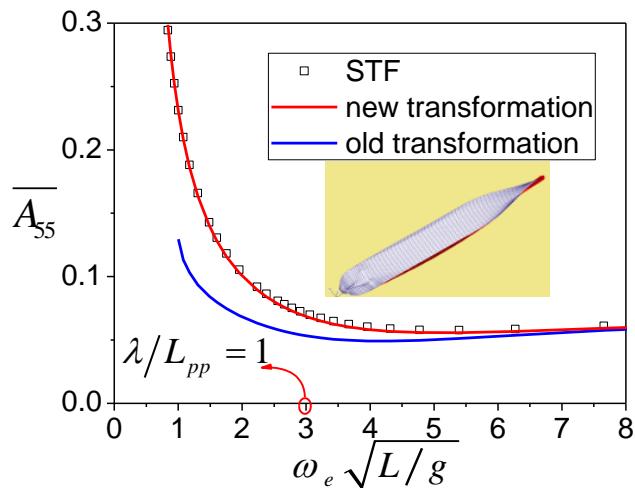
# The derivation of $c_{plus}$ term

$$A_{55} = A_{55}^0 + \frac{U^2}{\omega_e^2} A_{33}^0 = \mu_{55} - \frac{1}{\omega_e^2} c_{55} - \frac{1}{\omega_e} \int_0^\infty K_{55}(\tau) \cos \omega_e \tau d\tau$$

$$B_{55} = B_{55}^0 + \frac{U^2}{\omega_e^2} B_{33}^0 = b_{55} + \int_0^\infty K_{55}(\tau) \cos \omega_e \tau d\tau$$



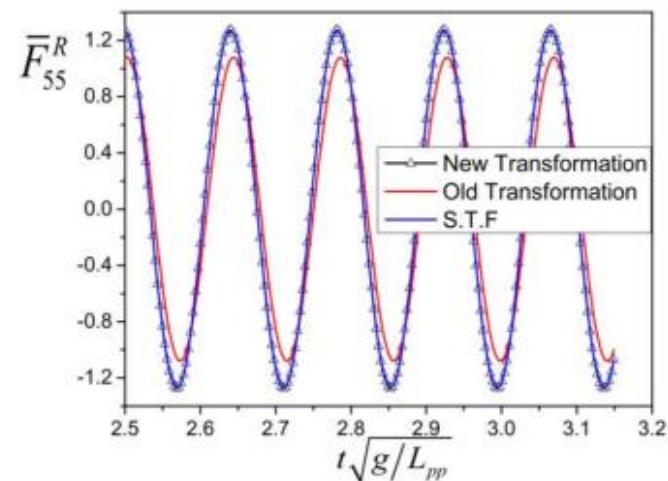
Physical model distortion



$$\bar{C}_{55} = c_{55} + c_{plus}$$

$$c_{plus} = -\frac{2U^2}{\pi} \int_0^\infty \frac{B_{33}(\omega_1)}{\omega_1^2} d\omega_1$$

Modification





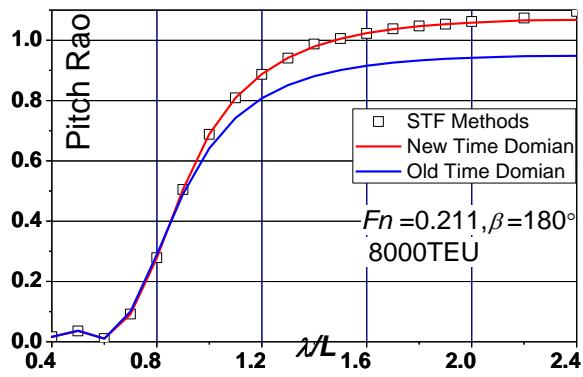
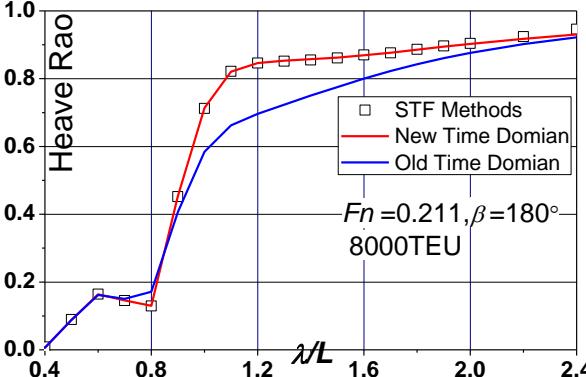
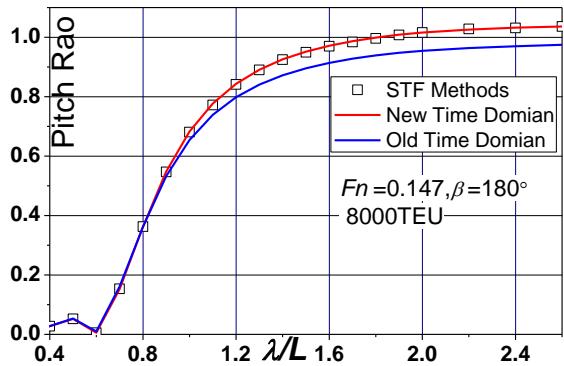
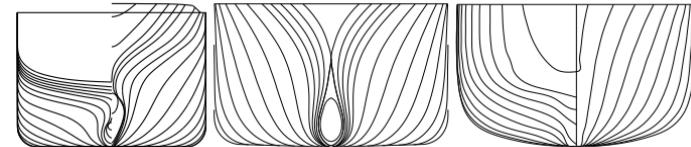
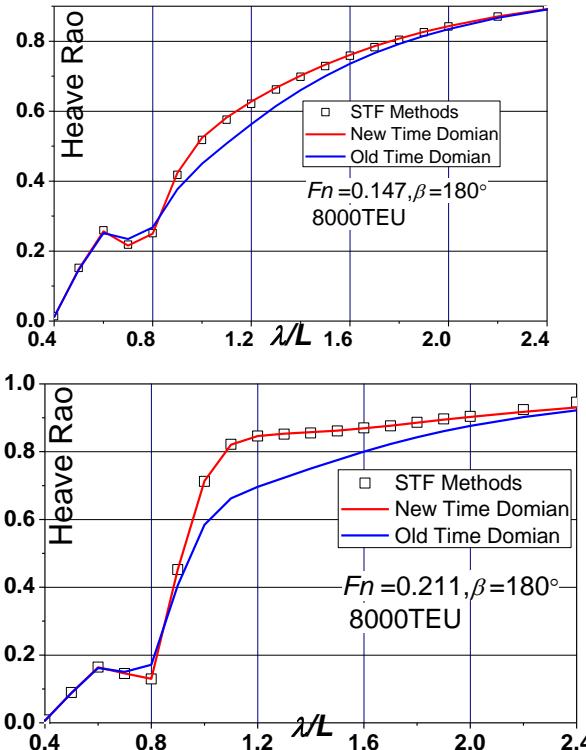
# Validation

	8000TEU	S175	Warship (model)
L <sub>pp</sub> (m)	319.92	175	6.157
B(m)	42.80	25.4	0.654
T(m)	13.00	9.5	0.207
▽(m <sup>3</sup> )	114067	24140	0.4022
CB	0.6408	0.572	
K <sub>yy`</sub>	0.25L <sub>pp</sub>	0.24L <sub>pp</sub>	0.25L <sub>pp</sub>

$F_n = 0.147$   
8000TEU

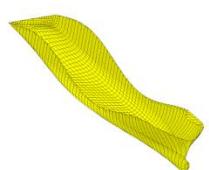
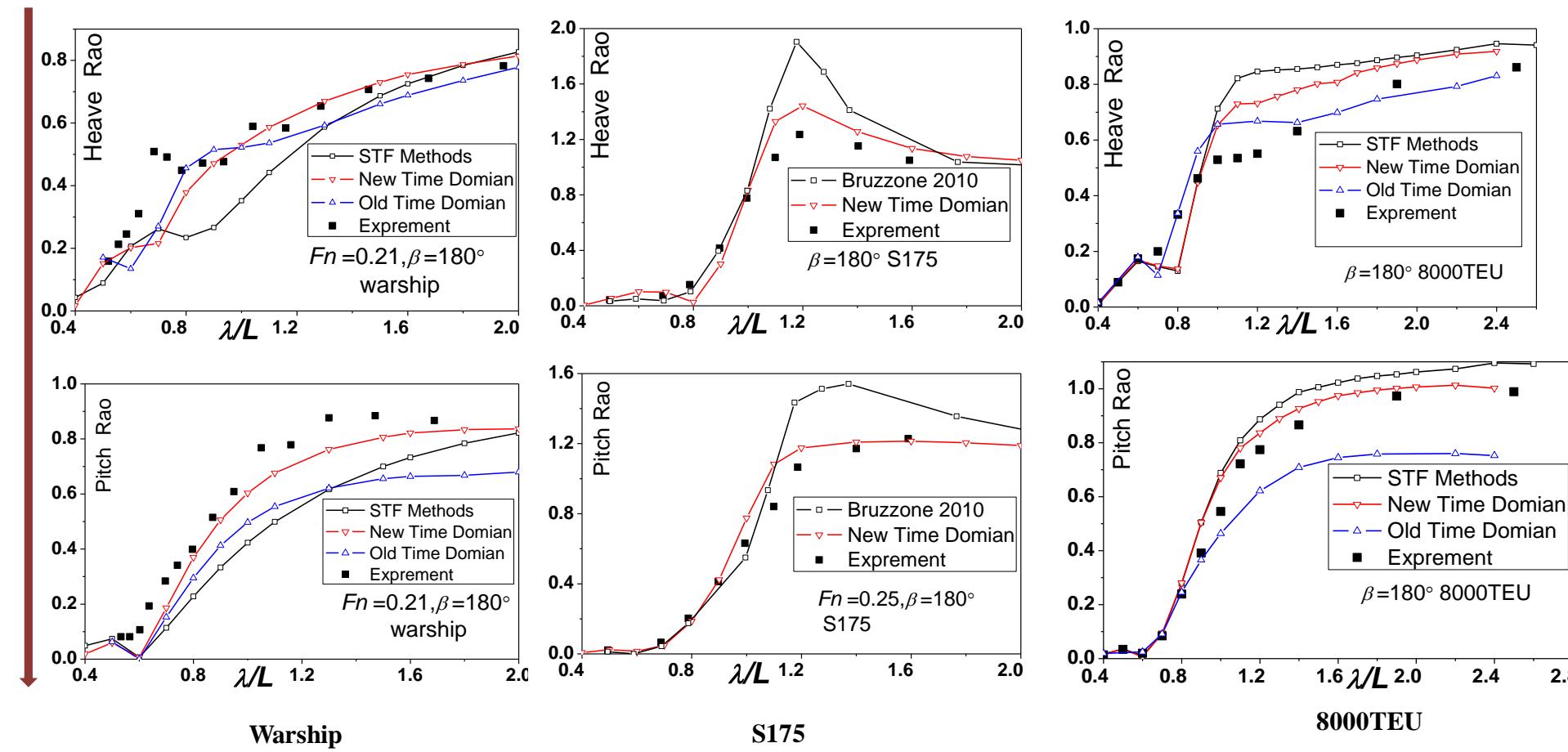
Here, radiation forces are represented by IRF in time domain

$F_n = 0.211$   
8000TEU





# An Enhanced Time Domain Strip Theory

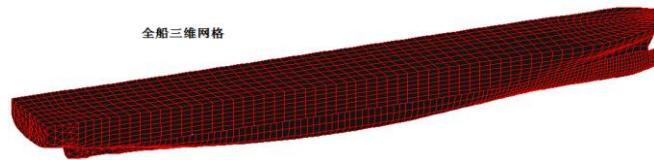


F-K forces calculated on instantaneous wet surface

# Numerical Simulation and Experiment Validation

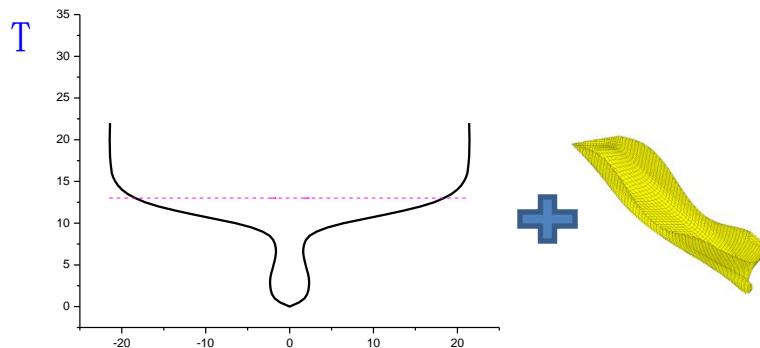


HEU Model Test



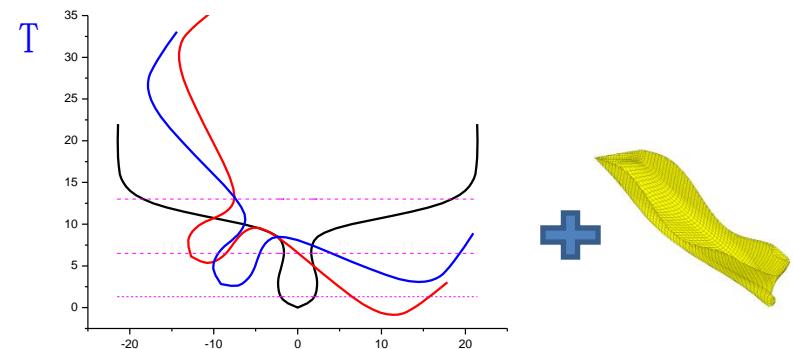
8000TEU Container Ship

Numerical model 1:



Linear radiation/diffraction force    Body-exact F-K force

Numerical model 2:



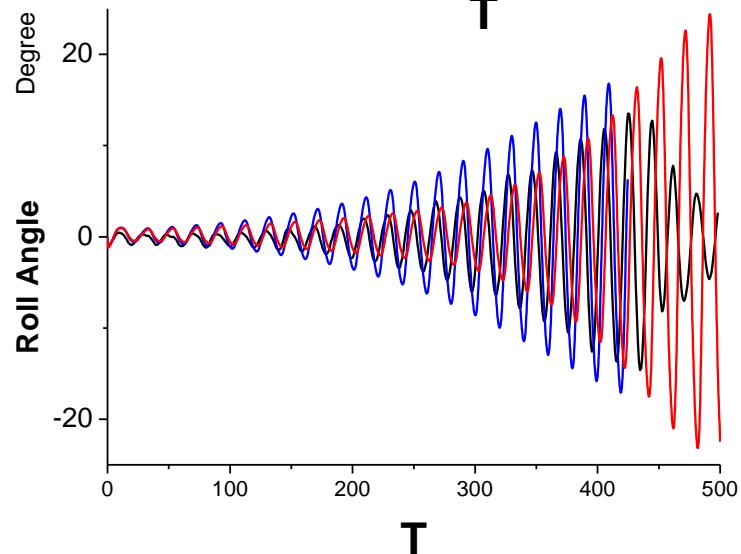
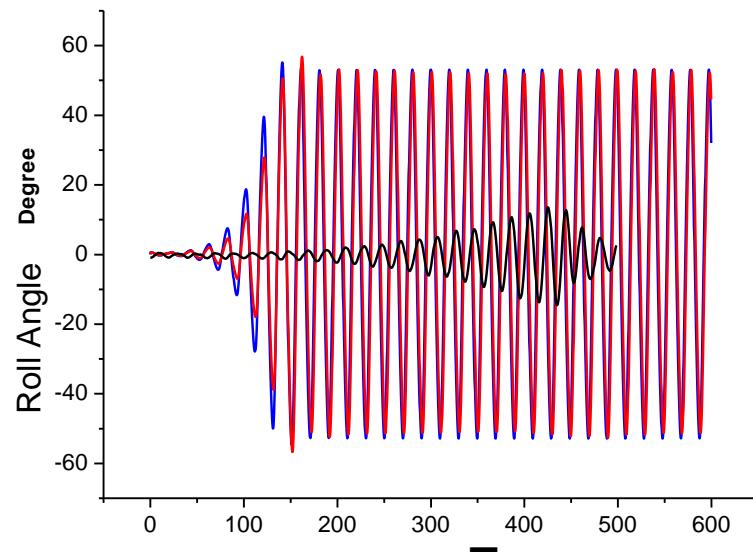
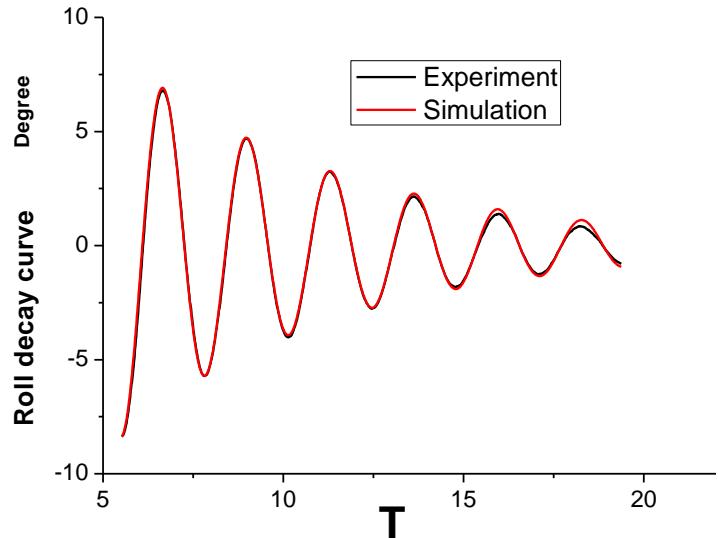
Body-exact radiation/diffraction force    Body-exact F-K force



# Numerical Simulation and Experiment Validation

$$\lambda / L = 1.04 \quad H / \lambda = 1/40 \quad Fn = 0.2$$

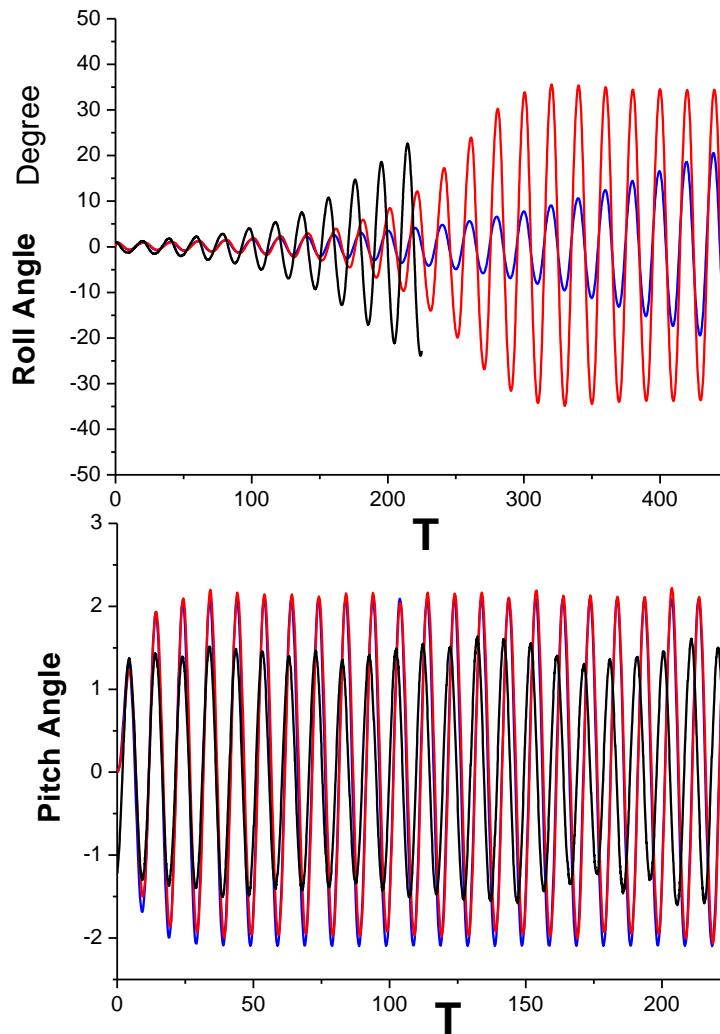
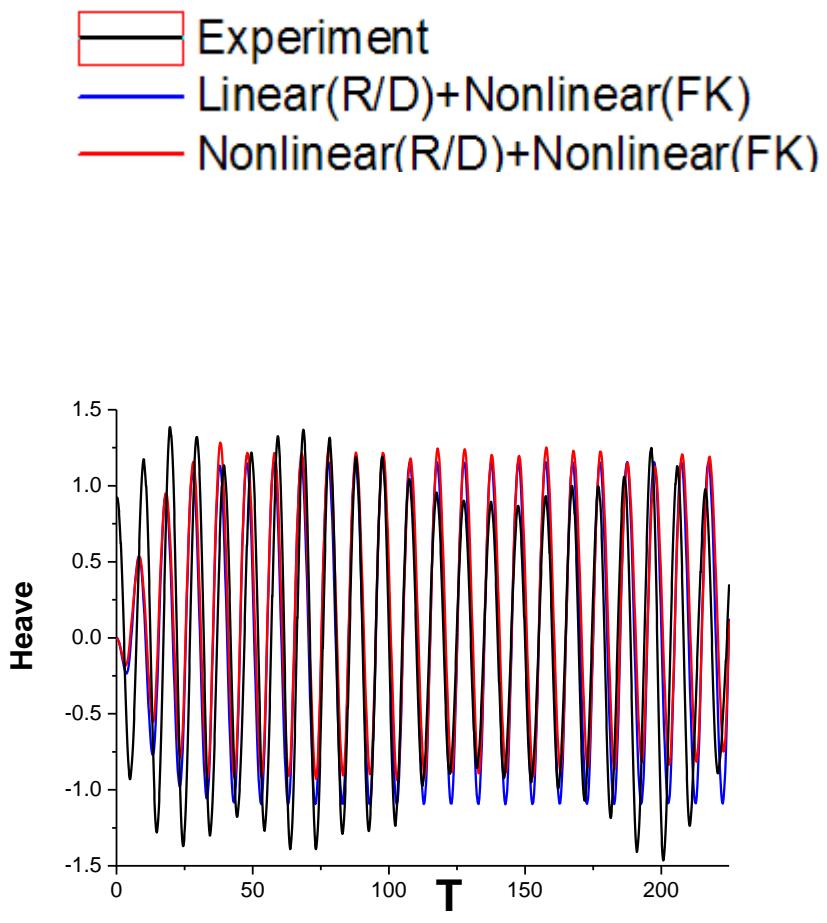
Experiment  
Linear(R/D)+Nonlinear(FK)  
Nonlinear(R/D)+Nonlinear(FK)





# Numerical Simulation and Experiment Validation

$$\lambda / L = 0.78 \quad H / \lambda = 1/40 \quad Fn = 0.1$$





# Conclusion

- ◆ Roll damping is very important on final steady roll amplitude
- ◆ Motion coupling may affect the developing of parametric rolling

## Further work

- ◆ Body-exact time domain hydrodynamics solver
- ◆ Methods to ascertain roll damping at large roll amplitude
- ◆ Efficient ways to evaluate parametric rolling in irregular waves